Mathematical Modeling of Cooperative Systems

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Cooperative systems, entities working together to achieve a common goal, are becoming important elements of our information society. What makes cooperative systems effective? What are the metrics for such a system? Can better structures and processes enhance cooperation and optimize systems? This presentation uses the emerging mathematical and game theory framework of subset team games, which provides a model for the principles, relationships, and metrics of cooperative phenomena. The subset team game framework of Arney and Peterson from cooperative game theory focuses on entities working together for a common good, reflecting the team-oriented cooperation that characterizes many systems and organizations. The underlying assumption of subset team games is that players (entities of the system or organization) are motivated by a combination of selfish and altruistic reasons, and the framework provides computable metrics from the system's utility function. This framework computes metrics from a carefully defined utility function for the system and its components. These metrics are important because system and organizational entities in real life are not rational in the strict sense of classical cooperative game theory. Instead, real cooperative entities are designed or choose to work together for the common good and thankfully humans often do not choose to follow the ruthless laws of economics.

In this presentation, we explain our use of mathematical modeling of system and organizational utility to understand teamwork. Our modeling framework includes cooperation space, which provides a visual means of assessing and comparing the cooperative nature of multiple algorithms geared toward the same tasks. Second, our cooperation complex provides a snapshot of the contributions of all subsets of a team. Together, these tools with their associated metrics provide a means of visualizing and comparing the cooperative nature of multiple systems or organizations. As examples, we illustrate their use in the movements and decisions of pursuit and evasion teams and the deployment location and movement of sensor/communication networks that require elements of geometric optimization to accomplish their missions. We compare several simulation algorithms and perform analysis of their performance and cooperation. While we understand an algorithm's rate of success is important, a singular focus on such a number can lead to disappointing results overall and possibly poor long-term performance by organizations. This is particularly true when cohesion and trust play key roles. Given the choice between multiple algorithms with equal success rates, it is better by far to choose a more altruistic algorithm since it has a more positive and longer-lasting impact on the system or organization.