Mathematical Modeling in Network Science

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Society's myriad networks connect not only humans, but also machines, computers, and robots. So how do these networks of complex relationships, collaborations, and communities of diverse entities work? This presentation uses a mathematical modeling framework to measure such networks' performances in several different ways. Traditional approaches to social network analysis are often performed either by studying individual nodes or by studying the graph as a whole. Beside statistical techniques, there are few theories that attempt a unified approach, encompassing both the individual node and the entire network. Other methods, such as clustering, allow investigation of only compositional properties. Our mathematical modeling framework implements these kinds of unified measures using a new cooperative game theory called subset team games. The framework of subset team games was developed to understand component cooperation and network performance simultaneously at the individual node and the entire network level.

Our modeling framework addresses several of the challenge issues put forth in the National Research Council's study on network science. Our models not only contribute to increasing the level of rigor and mathematical structure in networks, our work also has aspects associated with enabling the dynamics, spatial location, and information propagation in networks; modeling and analyzing of very large networks; designing and synthesizing of networks; and establishing better experiments and measurements of network structure. We report the results of applying our model to example networks to demonstrate progress on these challenges. In particular, we discuss how to decide which behaviors or algorithms are the most beneficial, cooperative and altruistic. We discuss methods for generating data in our framework's cooperation space, given a standard graph theoretic metric such as nodal centrality. We illustrate these metrics with examples that apply the techniques to specific networks, and discuss insights that can be gained from this approach. Finally, we discuss why altruism is a key factor in network effectiveness and the impact of these ideas on network disruption and identifying key nodes. We utilize our simulation results to measure system performance and node/channel properties of a communication network where the nodes are considered players in the network flow game and the general goal is to maximize flow through the network. Various cooperative/competitive flow management schemes are simulated and analyzed, looking in particular at the altruistic and selfish contributions of the players.