STATISTICAL MODELS FOR THE WIRELESS MULTIPATH RANDOM CHANNEL

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ABSTRACT: A theory developed for modeling communication channels as random processes [1] is used here to develop a statistical approach for characterizing propagation effects in wireless environments where multipath is a prevalent phenomenon affecting transmitted signals. This effort, which extends traditional methodologies [2] as well as work on signal detection algorithms in random channel propagation [3], models the amplitude scaling factors and path delays as random variables which in turn are used to evaluate statistical descriptors of the channel in the time domain (mean and autocorrelation function) and in the frequency domain by a function akin to the transfer function of a linear filter. Statistical models for the amplitude scaling factors and path delays are proposed and used to exemplify the procedure used to statistically characterize a multipath random channel.

A mathematical model often used to account for multipath propagation in a channel is, $y(t)=\sum_{n=1}^{N} a_n x(t-b_n)$, thereby implying that $h(t)=\sum_{n=1}^{N} a_n \delta(t-b_n)$, where $x(t)$, $y(t)$, and $h(t)$ are the input to, the output of, and the impulse response of the channel, respectively. The channel impulse response $h(t)$ does not account for the fact that amplitude scaling factors, delays, and even the number N of propagating paths cannot be predicted and therefore must be modeled as random variables. This inherently makes the impulse response $h(t)$ a random process parametrized by the amplitude scaling factors and delays, $\{a_n\}_{n=1}^{N}$ and $\{b_n\}_{n=1}^{N}$, respectively. In fact, a complete model would treat the amplitude scaling factors and delays as time dependent quantities, namely random processes, and the number of paths as a random variable. Such model even though quite complete is generally mathematically intractable from the point of view of statistically characterizing the channel. Therefore in the work herein, based on $h(t)$ as given, the channel will be modeled by the form

$$H(t)=\sum_{n=1}^{N} A_n \delta(t-B_n) \tag{1}$$

where the scaling factors $\{A_n\}_{n=1}^{N}$ and the delays $\{B_n\}_{n=1}^{N}$ form the random vectors $\underline{A}$ and $\underline{B}$, respectively, and the number of paths N will be modeled as being deterministic. The channel will therefore be statistically characterized in terms of the mean and the autocorrelation function of the impulse response, [1]. Additionally, efforts will be made to statistically characterize the channel output random process $Y(t)$ in terms of statistical knowledge of the input random process $X(t)$ and the channel impulse response $H(t)$. Throughout this work it will be assumed that $X(t)$ and $H(t)$ are statistically independent random processes. The following notation will be used,

$$E\{X(t)\}=m_X(t)=\text{mean of the input random process} \tag{2}$$
$$E\{X(t_1)X(t_2)\}=R_{XX}(t_1,t_2)=\text{autocorrelation function of the input random process} \tag{3}$$
$$E\{H(t)\}=m_H(t)=\text{mean of the channel impulse response} \tag{4}$$
$$E\{H(t_1)H(t_2)\}=R_{HH}(t_1,t_2)=\text{autocorrelation function of the channel impulse response} \tag{5}$$
$$E\{Y(t)\}=m_Y(t)=\text{mean of the output random process} \tag{6}$$
$$E\{Y(t_1)Y(t_2)\}=R_{YY}(t_1,t_2)=\text{autocorrelation function of the output random process} \tag{7}$$

The key relationship that allows statistical characterization of $Y(t)$ in terms of the input and the system impulse response is the well-known convolution integral $Y(t)=\int_{-\infty}^{\infty} H(\xi)X(t-\xi)d\xi$. For wide sense stationary (w.s.s) input processes where a description in terms of power spectral density (PSD) is possible, it is valuable to describe the output process in the frequency domain in terms of the input PSD and a channel frequency domain specification which will be shown equivalent to a transfer function for deterministic systems. There may be cases when the channel model must include a deterministic component as well as a random component. In such cases the expanded model is expressed as $H(t)=h_d(t)+H_r(t)$ and will be shown to not invalidate any of the analytical results to be developed. The addition of a deterministic component to the model modifies the channel statistics to the extent that as will be demonstrated, $h_d(t)$ contributes additional terms to the mean and the autocorrelation function of the expanded model. For the sake of conciseness, in most of the work presented, it will be assumed that the channel can be adequately modeled as having a random component only.

1. Statistics for the Random Channel Model

General results are first obtained on the mean and autocorrelation function of the channel impulse response of Eq. 1, where the random variables describing the channel amplitude scaling factors and time shifts are best described in vector form. Using capital letters to denote random variables, the following vectors are defined,

$$\underline{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_N \end{bmatrix}^T ;\ \underline{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_N \end{bmatrix}^T ;\ \underline{B} = \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix}^T ;\ \underline{b} = \begin{bmatrix} b_1 & b_2 & \cdots & b_N \end{bmatrix}^T \tag{1.1}$$

so that the channel is fully described from knowledge of the joint pdf $f_{\underline{A}\underline{B}}(\underline{a},\underline{b})$. Whereas other channel models can be analyzed, such as a low pass type with random DC gain and random cutoff frequency, the focus here is on the general model of Eq. 1. The channel statistics described by Eqs. 4 and 5 are first obtained as follows,

$$E\{H(t)\} = E\{\sum_{n=1}^{N} A_n \delta(t - B_n)\} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{n=1}^{N} a_n \delta(t - b_n) f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) d\underline{b} d\underline{a} \tag{1.2}$$

The integral of Eq. 1.2 is *2N*-fold and must be carried out term for term. When evaluating the *kth* term of the sum, all integrations of the elements of random vectors $\underline{A}$ and $\underline{B}$ will yield 1 for all r.v.'s except when the *kth* component is involved. Therefore obtaining a closed form expression for Eq. 1.2 involves the evaluation of

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_k \delta(t - b_k) f_{A_k B_k}(a_k, b_k) db_k da_k = \int_{-\infty}^{\infty} a_k f_{A_k B_k}(a_k, t) da_k \quad \Rightarrow \quad E\{H(t)\} = \sum_{n=1}^{N} \int_{-\infty}^{\infty} a_n f_{A_n B_n}(a_n, t) da_n \tag{1.3}$$

If for all N paths the random variables are pairwise statistically independent, so that for $k = 1, 2, \cdots, N$ $f_{A_k B_k}(a_k, b_k) = f_{A_k}(a_k) f_{B_k}(b_k)$, then Eq. 1.3 becomes,

$$E\{H(t)\} = \sum_{n=1}^{N} E\{A_n\} f_{B_n}(t) \tag{1.4}$$

Developing now the autcorrelation function, obtain

$$\begin{aligned} E\{H(t_1)H(t_2)\} &= E\{\sum_{n=1}^{N} A_n \delta(t_1 - B_n) \sum_{m=1}^{N} A_m \delta(t_2 - B_m)\} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{n=1}^{N} a_n \delta(t_1 - b_n) \sum_{m=1}^{N} a_m \delta(t_2 - b_m) f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) d\underline{b} d\underline{a} \\ &= \sum_{n=1}^{N}\sum_{m=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} a_n a_m \delta(t_1 - b_n) \delta(t_2 - b_m) f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) d\underline{b} d\underline{a} \\ &= \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} a_n^2 \delta(t_1 - b_n) \delta(t_2 - b_n) f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) d\underline{b} d\underline{a} + \sum_{n=1}^{N}\sum_{m'=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} a_n a_{m'} \delta(t_1 - b_n) \delta(t_2 - b_{m'}) f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) d\underline{b} d\underline{a} \end{aligned} \tag{1.5}$$

Note that the first multiple integral in Eq. 1.5 is *2N*-fold while the second is $2N(2N-1)$ fold. Henceforth, use of *m'* in place of *m* implies in the double sum that *m'* ≠ *n*. Evaluation of Eq. 1.5 cannot proceed without knowledge of the joint pdf of random vectors $\underline{A}$ and $\underline{B}$. Even if $\underline{A}$ and $\underline{B}$ are statistically independent, there can be dependence amongst the elements of each vector. For the case where $f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) = f_{\underline{A}}(\underline{a}) f_{\underline{B}}(\underline{b})$, Eq. 1.5 becomes

$$\begin{aligned} E\{H(t_1)H(t_2)\} &= \sum_{n=1}^{N} \int_{-\infty}^{\infty} a_n^2 f_{\underline{A}}(\underline{a}) d\underline{a} \int_{-\infty}^{\infty} \delta(t_1 - b_n)\delta(t_2 - b_n) f_{\underline{B}}(\underline{b}) d\underline{b} + \sum_{n=1}^{N}\sum_{m'=1}^{N} \int_{-\infty}^{\infty} a_n a_{m'} f_{\underline{A}}(\underline{a}) d\underline{a} \int_{-\infty}^{\infty} \delta(t_1 - b_n)\delta(t_2 - b_{m'}) f_{\underline{B}}(\underline{b}) d\underline{b} \\ &= \sum_{n=1}^{N} E\{A_n^2\} \delta(t_1 - t_2) f_{B_n}(t_2) + \sum_{n=1}^{N}\sum_{m'=1}^{N} \int_{-\infty}^{\infty} a_n a_{m'} f_{\underline{A}}(\underline{a}) d\underline{a} \int_{-\infty}^{\infty} \delta(t_1 - b_n)\delta(t_2 - b_{m'}) f_{\underline{B}}(\underline{b}) d\underline{b} \end{aligned} \tag{1.6}$$

where simplification in Eq. 1.6 is possible because each integral is now N-fold since the pdf's over which the integrations are carried out are N dimensional. A simple, fairly realistic case is one where each random vector consists of statistically independent r.v.'s, so that the joint pdf of the two random vectors further simplifies to $f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) = \prod_{n=1}^{N} f_{A_n}(a_n) f_{B_n}(b_n)$. With all $2N$ r.v.'s statistically independent, the second term in Eq. 1.6 can be further simplified so that

$$E\{H(t_1)H(t_2)\} = \sum_{n=1}^{N} E\{A_n^2\}\delta(t_1 - t_2) f_{B_n}(t_2) + \sum_{n=1}^{N}\sum_{m'=1}^{N} E\{A_n\}E\{A_{m'}\} f_{B_n}(t_1) f_{B_{m'}}(t_2) \tag{1.7}$$

However, a more realistic case involves amplitude and delay variables are statistically related (i.e., correlated).

When a channel is modeled by the form $H(t) = h_d(t) + H_r(t)$, the statistics developed above are applicable to the random component of the channel model. The effect of a deterministic component on a channel model is of simple additive form on the mean of $H(t)$, namely $E\{H(t)\} = h_d(t) + E\{H_r(t)\} = h_d(t) + m_{H_r}(t)$ and of additive/multiplicative form on the autocorrelation function, $E\{H(t_1)H(t_2)\} = R_{HH}(t_1,t_2) = E\{[h_d(t_1) + H_r(t_1)][h_d(t_2) + H_r(t_2)]\} = h_d(t_1)h_d(t_2) + h_d(t_1)m_{H_r}(t_2)$ $+ h_d(t_2)m_{H_r}(t_1) + E\{H_r(t_1)H_r(t_2)\}$. The results on channel autocorrelation function obtained above for the model of Eq. 1 under various assumptions of statistical independence of random variables are applicable to the last term $E\{H_r(t_1)H_r(t_2)\}$ of $E\{H(t_1)H(t_2)\}$. Observe however that the time average of this autocorrelation function simplifies the results obtained because any reasonable deterministic impulse response component to the channel impulse response is an "energy type" time function with zero time average. Similarly, the time average of the mean function of the random component of the channel impulse response under reasonable circumstances can be expected to have a zero time average. This then yields, $\langle R_{HH}(t+\tau,t)\rangle = \langle E\{H_r(t+\tau)H_r(t)\}\rangle$ thereby demonstrating that the presence of a deterministic component in the channel impulse response model affects the channel mean but not its time averaged autocorrelation function.

2. General Results on the Output Statistics

Here, the statistics of the channel output process $Y(t)$ are presented without derivation as the steps necessary to obtain the stated results may be found in [1]. These results are later used to analyze specific cases involving the channel model introduced in Section 1 with statistics as developed therein. First, the output mean is given by

$$E\{Y(t)\} = E\left\{\int_{-\infty}^{\infty} H(\xi)X(t-\xi)d\xi\right\} = \int_{-\infty}^{\infty} E\{H(\xi)X(t-\xi)\}d\xi = \int_{-\infty}^{\infty} E\{H(\xi)\}E\{X(t-\xi)\}d\xi \tag{2.1}$$

The simplification of the expectation operation inside the integral is made possible by the fact that the channel and the input, modeled as random processes, are assumed to be statistically independent. When the input process additionally is w.s.s. so that the expectation $E\{X(t-\xi)\}$ is constant, say m_X, then

$$E\{Y(t)\} = m_X \int_{-\infty}^{\infty} E\{H(\xi)\}d\xi = m_X \int_{-\infty}^{\infty} m_H(\xi)d\xi \tag{2.2}$$

where the notation of Eq. 4 has been used in Eq. 2.2. This result also points to the fact that "reasonable" channels are nonstationary processes. Otherwise, the above result would yield an unbounded output mean, as with $H(t)$ a w.s.s. process, $E\{H(t)\} = m_H$ and the integral of Eq. 2.2 diverges. Now, without showing the derivation, obtain

$$E\{Y(t_1)Y(t_2)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)R_{XX}(t_1-\xi,t_2-\eta)d\xi d\eta = R_{YY}(t_1,t_2) \tag{2.3}$$

When the input process is w.s.s., then $R_{XX}(t_1-\xi,t_2-\eta) = R_{XX}(t_1-t_2-(\xi-\eta))$, so that Eq. 2.3 becomes

$$E\{Y(t_1)Y(t_2)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)R_{XX}(t_1-t_2-(\xi-\eta))d\xi d\eta = R_{YY}(t_1-t_2) \tag{2.4}$$

The results of Eqs. 2.2 and 2.4 demonstrate that when the channel input is a w.s.s. random process, so is the channel output process. Even thought the channel when modeled as a random process is inherently nonstationary, the stationarity of the output is consistent with classical results, [4]. Now, with $t_1 = t + \tau$ and $t_2 = t$, Eq. 2.4 can be expressed in the form

$$E\{Y(t+\tau)Y(t)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)R_{XX}(t+\tau-\xi,t-\eta)d\xi d\eta = R_{YY}(t+\tau,t) \qquad (2.5)$$

so that applying a time averaging operation for which the notation $\langle \bullet \rangle = \lim_{T\to\infty} T^{-1}\int_{-T/2}^{T/2} \bullet dt$ is used, obtain

$$\bar{R}_{YY}(\tau) = \langle R_{YY}(t+\tau,t)\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)\langle R_{XX}(t+\tau-\xi,t-\eta)\rangle d\xi d\eta \qquad (2.6)$$

When the input process is w.s.s., time averages are not necessary, so that Eq. 2.5 becomes

$$E\{Y(t+\tau)Y(t)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)R_{XX}(\tau-(\xi-\eta))d\xi d\eta = R_{YY}(\tau) \qquad (2.7)$$

Using $\Im\{\bullet\}$ to denote both one or two-dimensional Fourier Transforms as appropriate, then $\Im\{R_{XX}(\tau)\} = S_{XX}(f)$ and $\Im\{R_{YY}(\tau)\} = S_{YY}(f)$ are the PSD's of the input and output processes, respectively, so that from Eq. 2.7

$$S_{YY}(f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)\Im\{R_{YY}(\tau-(\xi-\eta))\}d\xi d\eta \qquad (2.8)$$

Since $\Im\{R_{XX}(\tau-(\xi-\eta))\} = S_{XX}(f)e^{-j2\pi f(\xi-\eta)}$, Eq. 2.8 yields

$$S_{YY}(f) = S_{XX}(f)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)e^{-j2\pi f(\xi-\eta)}d\xi d\eta = S_{XX}(f)S_{HH}^{d}(f,f) \qquad (2.9)$$

where the double Fourier Transform

$$S_{HH}^{d}(f,\nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{HH}(\xi,\eta)e^{-j2\pi(f\xi-\nu\eta)}d\xi d\eta \qquad (2.10)$$

has been used in order to obtain the compact result of Eq. 2.9. For the more general case of Eq. 2.6, from

$$\langle R_{XX}(t+\tau-\xi,t-\eta)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} R_{XX}(t+\tau-\xi,t-\eta)dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2-\eta}^{T/2-\eta} R_{XX}(\lambda+\tau-(\xi-\eta),\lambda)d\lambda \qquad (2.11)$$

and labeling the time averaged autocorrelation function of Eq. 2.11 $\bar{R}_{XX}(\tau-(\xi-\eta))$, the more general case produces a result essentially identical to that of Eq. 2.9 except that $S_{XX}(f)$ and $S_{YY}(f)$ are replaced by $\bar{S}_{XX}(f) = \Im\{\bar{R}_{XX}(\tau)\}$, and $\bar{S}_{YY}(f) = \Im\{\bar{R}_{YY}(\tau)\}$, respectively. In Eq. 2.9, $S_{HH}^{d}(f,f)$ which can be shown to be real, even, and nonnegative when $R_{HH}(t_1,t_2)$ is real, plays the role of the magnitude squared of the channel transfer function in theory.

3. Statistical Models for Channel Amplitude and Delay Components

The channel specification of Eq. 2 is dependent on knowledge of the statistics of the amplitude scaling factors and delay components which in this work are modeled as random vectors $\underline{A}$ and $\underline{B}$, respectively. Such specification involves the joint pdf of the random vectors, namely $f_{\underline{A}\underline{B}}(\underline{a},\underline{b})$. However few such joint pdf's are readily available to be applied to the statistical characterization of the channel. In this section, characterization of the amplitude scaling factors and delays in terms of joint pdf's are proposed and key features described. The basic simplifying assumption that is made involves the pairwise statistical independence of the amplitudes and delays in the sense that $f_{\underline{A}\underline{B}}(\underline{a},\underline{b}) = \prod_{i=1}^{N} f_{A_iB_i}(a_i,b_i)$. Generally it is assumed the behavior of the amplitude and delay along one path is not affected by those of another path. However, over any one path there is some degree of dependence between amplitude and delay as path delay is directly related to distance between the transmitter and receiver. This in turn implies that generally this greater travel distance results in greater signal strength attenuation. A model for the pdf's $f_{A_iB_i}(a_i,b_i)$, $i=1,2,\cdots,N$ should reflect this relationship between the amplitude scaling factors and delays. Therefore, five different models for the joint pdf are proposed as shown in the table below.

	$f_{AB}(a,b)$ sub index i is dropped	$f_A(a)$	$f_B(b)$	$E\{AB\}$
1	$\sigma a e^{-(1+\sigma b)a}u(a)u(b)$	$e^{-a}u(a)$	$\frac{\sigma}{(1+\sigma b)^2}u(b)$	$\frac{1}{\sigma}$
2	$\sigma^2 b_0 a e^{-\sigma ab}u(a)u(b-b_0)$	$\sigma b_0 e^{-\sigma a b_0}u(a)$	$\frac{b_0 u(b-b_0)}{b^2}$	$\frac{2}{\sigma}$
3	$\frac{\sigma_a^2\sigma_b^2}{\sigma_a+\sigma_b}(a+b)e^{-(\sigma_a a+\sigma_b b)}u(a)u(b)$	$\frac{\sigma_a^2\sigma_b}{\sigma_a+\sigma_b}\left[a+\frac{1}{\sigma_b}\right]e^{-\sigma_a a}u(a)$	$\frac{\sigma_a\sigma_b^2}{\sigma_a+\sigma_b}\left[b+\frac{1}{\sigma_a}\right]e^{-\sigma_b b}u(b)$	$\frac{2}{\sigma_a\sigma_b}$
4	$abe^{-ab^2/\sqrt{2}b_0}u(a)u(b-b_0)$	$\frac{b_0 e^{-ab_0/\sqrt{2}}u(a)}{\sqrt{2}}$	$\frac{2b_0^2}{b^3}u(b-b_0)$	$\frac{2^{3/2}}{3}$
5	$2b_0^2 abe^{-ab^2}u(a)u(b-b_0)$	$b_0^2 e^{-ab_0^2}u(a)$	$\frac{2b_0^2 u(b-b_0)}{b^3}$	$\frac{2}{3b_0}$

For the five models presented, only the most salient features have been provided. Other parameters, such as mean, variance, conditional pdf, conditional mean and conditional variance of the random variables A and B have been determined in [5]. In all five cases there is some degree of correlation between the two random variables that is generally controlled by one or more parameters. These models vary in usefulness as they are not all equally good for modeling the behavior of wireless channels. Moreover some models may be mathematically intractable for determining the channel mean, autocorrelation, and output statistics. Specific results are obtained below as an example for model #3 above.

4. Example of Channel Statistics for a Specific Amplitude and Delay Model

Consider now evaluation of the channel mean and autocorrelation function as well as $S_{HH}^{d}(f,\nu)$ corresponding to

$$f_{A_nB_n}(a_n,b_n)=\frac{\sigma_a^2\sigma_b^2}{\sigma_a+\sigma_b}(a_n+b_n)e^{-(\sigma_a a_n+\sigma_b b_n)}u(a_n)u(b_n) \tag{4.1}$$

which is the third model in the above table specifying the joint pdf for the amplitude scaling factors and delay. The parameters σ_a and σ_b in Eq. 4.1 affecting all aspects of the two random variables (such as means, variances, etc.) are assumed the same for all paths, and therefore are not indexed to a path on the assumption that since all paths exist within the same channel, they have similar statistical characteristics. Thus from Eq. 1.3, obtain

$$E\{H(t)\}=\sum_{n=1}^{N}\int_{-\infty}^{\infty}a_n f_{A_nB_n}(a_n,t)da_n \sum_{n=1}^{N}\int_{-\infty}^{\infty}a_n\frac{\sigma_a^2\sigma_b^2}{\sigma_a+\sigma_b}(a_n+t)e^{-(\sigma_a a_n+\sigma_b t)}u(a_n)u(t)da_n \tag{4.2}$$

Since the integral of Eq. 4.2 independent of the index n , its simple evaluation yields

$$E\{H(t)\}=\sum_{n=1}^{N}\left[\frac{\sigma_b^2(2+\sigma_a t)}{\sigma_a^2+\sigma_a\sigma_b}\right]e^{-\sigma_b t}u(t)=N\left[\frac{\sigma_b^2(2+\sigma_a t)}{\sigma_a^2+\sigma_a\sigma_b}\right]e^{-\sigma_b t}u(t) \tag{4.3}$$

thus showing the channel mean decays as a function of time. From Eq. 1.5 the channel autocorrelation function becomes,

$$E\{H(t_1)H(t_2)\}=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\sum_{n=1}^{N}a_n\delta(t_1-b_n)\sum_{m=1}^{N}a_m\delta(t_2-b_n)f_{A_1\cdots A_NB_1\cdots B_N}(a_1,\cdots a_N,b_1,\cdots,b_N)db_1\cdots db_n da_1\cdots da_n \tag{4.4}$$

The integration is $2N$ - fold and there are N^2 terms resulting from the product of the two sums. For each term, a $2N$ -fold integration must be performed, leading to a result of 1 for all except one double integral that must be evaluated with $n = m$ and also with $n \neq m$. For $n = m$, a typical term (there are N such identical terms) yields,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_n^2 \delta(t_1 - b_n)\delta(t_2 - b_n) f_{A_nB_n}(a_n, b_n) da_n db_n = \int_{-\infty}^{\infty} a_n^2 \delta(t_1 - t_2) f_{A_nB_n}(a_n, t_2) da_n \qquad (4.5)$$

Using Eq. 4.1 in Eq. 4.5 yields $\frac{\sigma_b^2}{\sigma_a + \sigma_b}\left[\frac{6}{\sigma_a^2} + \frac{2t_2}{\sigma_a}\right] e^{-\sigma_b t_2} \delta(t_1 - t_2) u(t_2)$ for $n = m$. For $n \neq m$, a typical term yields,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_n a_m \delta(t_1 - b_n)\delta(t_2 - b_m) f_{A_nA_mB_nB_m}(a_n, a_m, b_n, b_m) db_n db_m da_n da_m = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_n a_m f_{A_nB_n}(a_n, t_1) f_{A_mB_m}(a_m, t_2) da_n da_m \qquad (4.6)$$

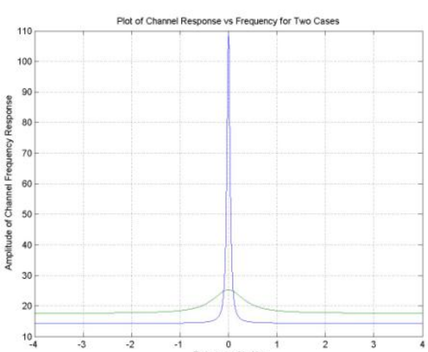
where there are $N^2 - N$ such terms. The simplification in Eq. 4.6 is obtained by integrating over the inner variables and applying the assumed pair wise independence condition. Leaving out the tedious steps needed for evaluating the remaining double integral in Eq. 4.6, the result obtained is $\frac{\sigma_b^2}{\sigma_a + \sigma_b}\left[\frac{2}{\sigma_a} + t_1\right] e^{-\sigma_b t_1} u(t_1) \frac{\sigma_b^2}{\sigma_a + \sigma_b}\left[\frac{2}{\sigma_a} + t_2\right] e^{-\sigma_b t_2} u(t_2)$ for $n \neq m$. Thus,

$$R_{HH}(t_1, t_2) = N \frac{\sigma_b^2}{\sigma_a + \sigma_b}\left[\frac{6}{\sigma_a^2} + \frac{2t_2}{\sigma_a}\right] e^{-\sigma_b t_2} \delta(t_1 - t_2) u(t_2) + (N^2 - N)\frac{\sigma_b^4}{(\sigma_a + \sigma_b)^2}\left[\frac{2}{\sigma_a} + t_1\right] e^{-\sigma_b t_1} u(t_1)\left[\frac{2}{\sigma_a} + t_2\right] e^{-\sigma_b t_2} u(t_2) \qquad (4.7)$$

Obtain now the frequency domain characterization of the channel via $S_{HH}^d(f, \nu)$. Using Eqs. 2.10 and 4.7, obtain

$$S_{HH}^d(f, \nu) = \frac{N\sigma_b^2}{\sigma_a^2(\sigma_a + \sigma_b)}\left[\frac{6}{\sigma_b + j2\pi(f - \nu)} + \frac{2\sigma_a}{(\sigma_b + j2\pi(f - \nu))^2}\right] + \frac{N^2 - N}{\sigma_a^2}\frac{\sigma_b^4}{(\sigma_a + \sigma_b)^2}\left[\frac{2}{\sigma_b + j2\pi f} + \frac{\sigma_a}{(\sigma_b + j2\pi f)^2}\right]$$
$$\times\left[\frac{2}{\sigma_b - j2\pi\nu} + \frac{\sigma_a}{(\sigma_b - j2\pi\nu)^2}\right] \Rightarrow S_{HH}^d(f, f) = \frac{2N(3\sigma_b + \sigma_a)}{\sigma_a^2(\sigma_a + \sigma_b)} + \frac{N^2 - N}{\sigma_a^2}\frac{\sigma_b^4}{(\sigma_a + \sigma_b)^2}\left[\frac{(\sigma_a + 2\sigma_b)^2 + (4\pi f)^2}{(\sigma_b^2 + (2\pi f)^2)^2}\right] \qquad (4.8)$$

where the result of Eq. 4.8 follows from $S_{HH}^d(f, \nu)$ with $\nu = f$. A plot of $S_{HH}^d(f, \nu)$ for a few representative values of the parameters N , σ_a , and σ_b is shown below , with $N = 4$ and $\sigma_a = 1/2$, $\sigma_b = 1/3$ and $\sigma_a = 2$, $\sigma_b = 3$.



Conclusions

Random channel modeling is used to obtain a statistical characterization in terms of mean, autocorrelation function as well as in the frequency domain via a function akin to a transfer function for a multipath channel in wireless communication applications. The results obtained are consistent with well-established theory but provide an additional framework for modeling and analyzing communication channels in terms of their effect on the performance of wireless systems.

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