Inquiry-Based 5E Lesson in a High School Geometry Classroom: The Rediscovery and Applications of the Golden Ratio

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Synopsis**:**

The 5E inquiry model of instruction is a constructivist teaching methodology that allows students to develop their own pathways to understanding of novel concepts. In this lesson, high school geometry students explore the golden ratio of phi. Students discover and derive the golden ratio through a guided inquiry activity. This methodology allows students to become immersed in the subject matter while creating their own explanations of mathematical concepts.

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Introduction

5-E Learning Cycle

The 5-E learning cycle is a product of the constructivist learning theory. In a constructivist classroom, the teacher assumes students enter with the unique experiences and beliefs about how the world works (i.e. scientific ideas) based upon those experiences (Colburn, 2000 – "Constructivism: science education's 'grand unifying theory'."). The 5-2 learning cycle is an inquiry approach to understanding science and mathematical concepts.
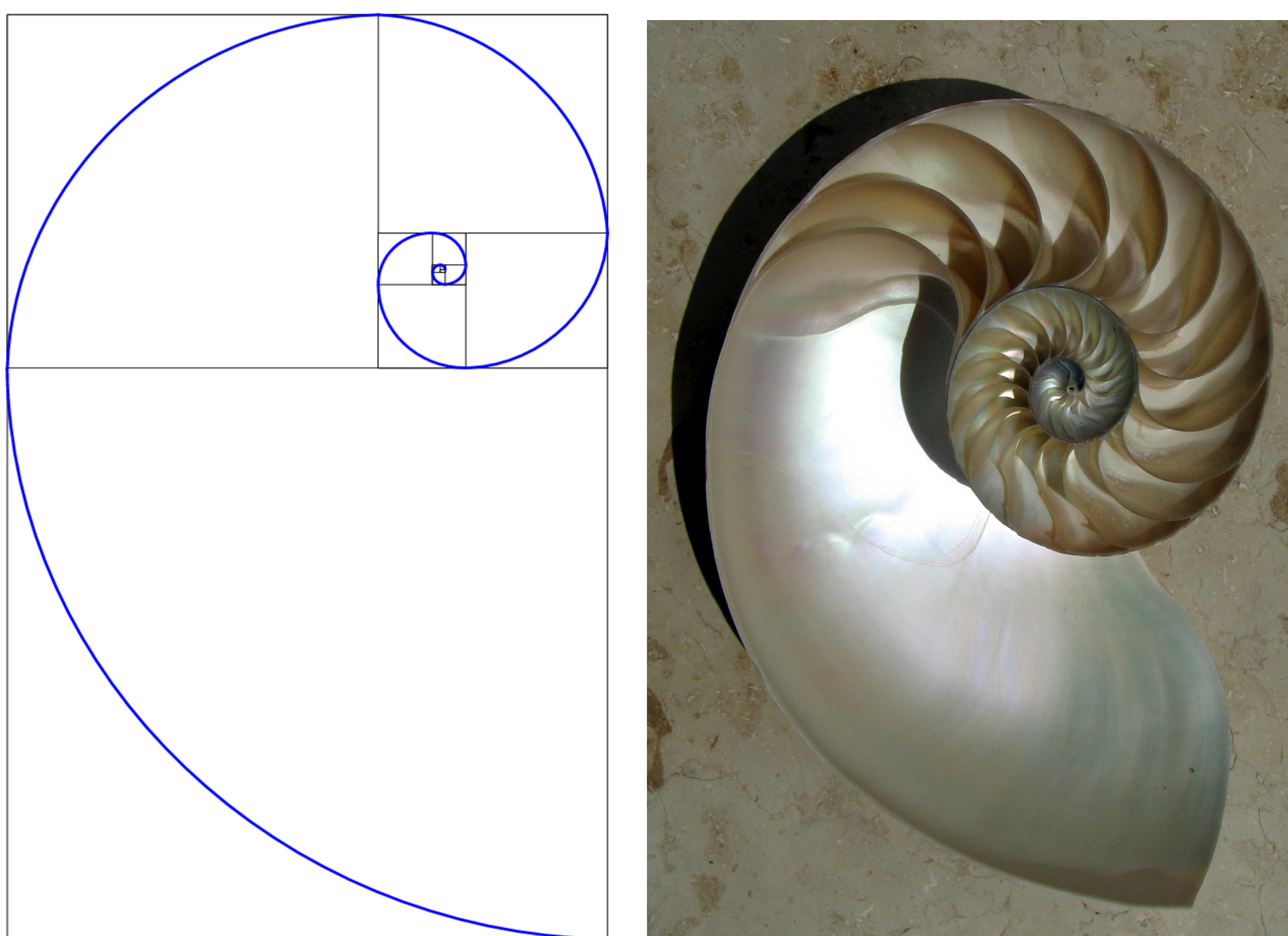

Objectives and Standards

- Students will be able to use prior knowledge of similar figures and the quadratic formula to find the unique irrational number of phi (the golden ratio).

NCTM Geometry Standard

- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
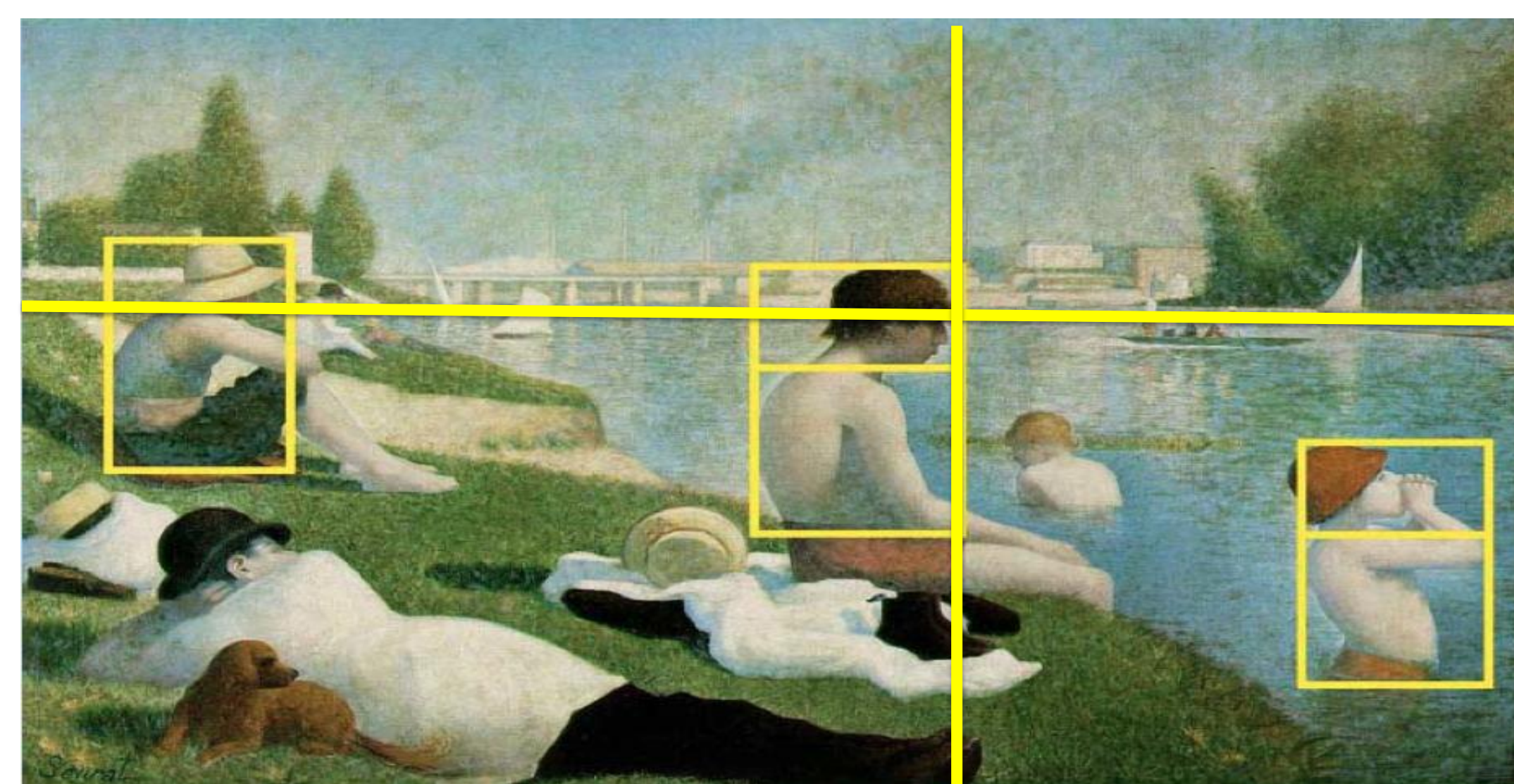
Standards for Mathematical Practice (CCSS)

- Look For and Make Use of Structure

Lesson Plan

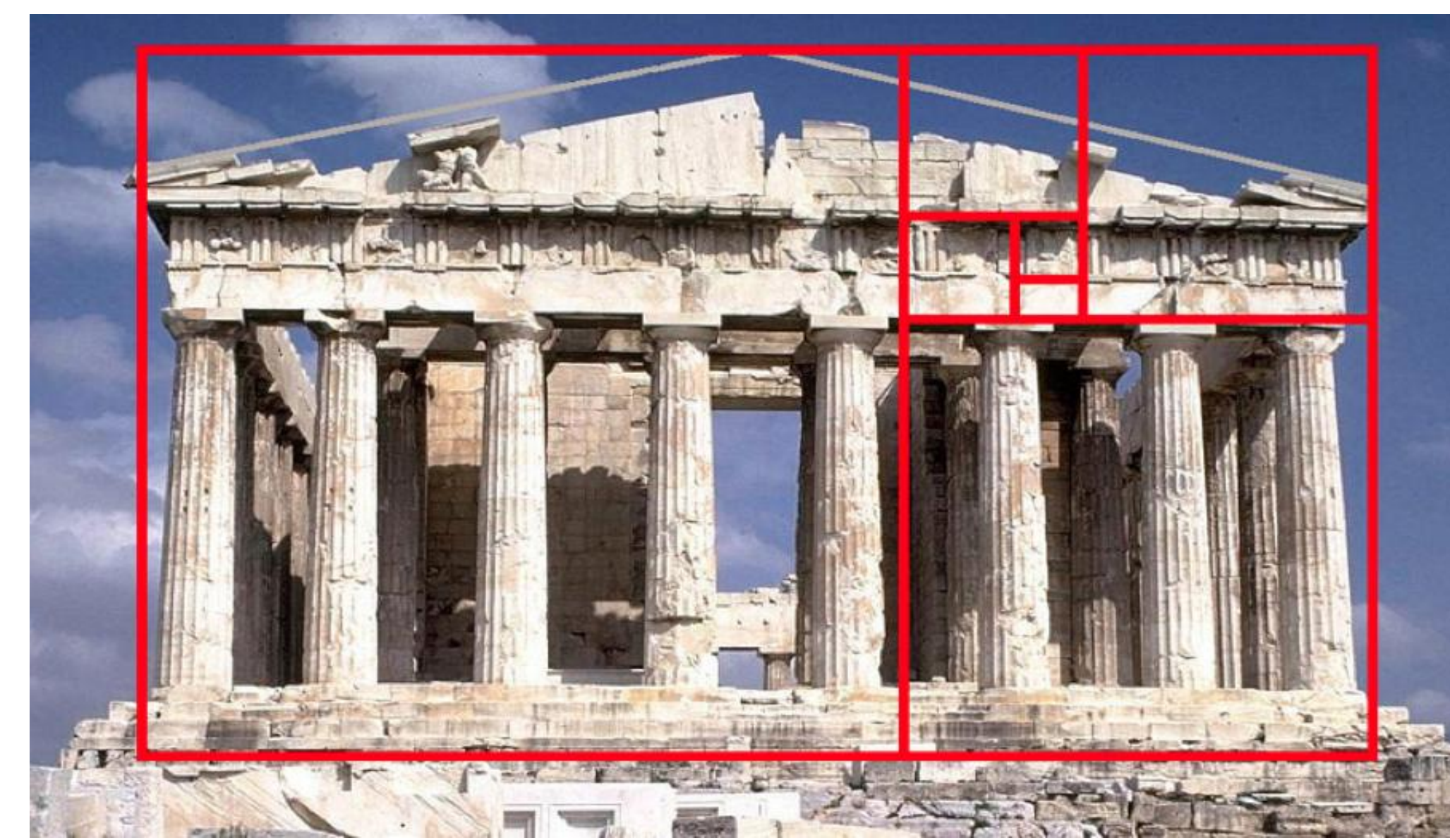|	
Engagement	Students will be shown famous paintings such as Da Vinci's *Mona Lisa* and *The Last Supper*, Seurat's *Bathers at Asnieres,* and Raphael's *Crucifixion*. Arousing interest, the students will vote on a series of rectangles and be asked which they believe is the most aesthetically pleasing.
Exploration	Students are rediscovering the golden ratio, phi, through the use of similar figures and the quadratic formula. Students measures dimensions of a divided rectangle to create a similarity proportion. Students then use the quadratic formula to produce the golden ratio.
Explanation	The students scaffold what they have just discovered. Through prior knowledge, students rediscover and affirm their prior knowledge of a very unique irrational number. The teacher will reveal the mystery behind the paintings shown during the engagement.
Elaboration	Students deepen their understanding of the golden ratio in novel situations where students must apply their newfound knowledge. Students utilize the ratio to construct rectangles based on the proportion of phi. Once students create a golden spiral, the teacher will present the class with applications in the world where the spiral is found.
Evaluation	Evaluation occurs through informal questioning and observation throughout the lesson.

Prior Knowledge

Students can:
- Determine Similarity and use proportions to find missing lengths
- Solve a quadratic equation using the quadratic formula

STEAM Connection

Though this a mathematics lesson, the artistic and visual elements the golden ratio provide appeal to artists and art education.

If you ignore the text and images on this poster, which size of the bold, blue rectangles draws your attention the most?

Summary

The 5E inquiry model of instruction is a constructivist teaching methodology that allows students to develop their own pathways to understanding of novel concepts. In this lesson, high school geometry students explore the golden ratio of phi. Students discover and derive the golden ratio through a guided inquiry activity. This methodology allows students to become immersed in the subject matter while creating their own explanations of mathematical concepts.

The Golden Ratio

Students are shown a series of rectangles in a single series. The teachers asks the students which rectangle their eyes were drawn to first and what rectangle they find the most aesthetically pleasing to the eye.

By dividing a rectangle to create two similar rectangles, students must rely on their prior knowledge of similar shapes and solving proportions to find the the value of x.

$$\frac{x}{1} = \frac{1}{x-1}$$

Using the quadratic formula to solve the proportion,

$$1 = x^2 - x$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 + \sqrt{5}}{2} = \varphi \approx 1.618\ldots$$

The Golden Spiral

Students start by positioning a 1x1 square on a piece of graph paper. Students utilize the Fibonacci sequence to build the golden spiral off the original square to create interconnected golden rectangles. Students will finish the spiral by drawing the arc from the corner of the rectangle to the segment in the opposite corner, which creates the larger golden rectangle.

Fibonacci Sequence	Rectangle Added
1	1x1
1	1x1
2	2x2
3	3x3
5	5x5
8	8x8
13	13x13

1.6180339887498948482045868343656381177203091798057628621354486227052604628189024497072072041893911374847540880753868917521266338622235369317931800607667263544331…