

Neighborhood Homogeneous Labelings of Graphs

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Abstract

Graphs in which all vertices have the same degree (regular graphs) can be thought of as being homogeneous – all vertices “look the same” from the perspective of vertex degree. We consider a more restrictive kind of homogeneity, utilizing vertex and edge labels. Given a labeling of the vertices and edges of a graph, we require that the neighborhood of every vertex contains the same number of each of the labels. This homogeneity constraint is a generalization of regularity – all such graphs are regular. We consider a specific homogeneity condition in which both the edge and vertex label sets have two elements and every neighborhood contains two of each label. We show that vertex homogeneity implies edge homogeneity (so long as the number of edges in any neighborhood is four), and give two theorems describing how to build new homogeneous graphs (or multi-graphs) from others. This gives a characterization of these neighborhood homogeneous graphs in terms of explicit constructions. We discuss the genesis of these ideas from the problem of detecting inhomogeneities in communications graphs, and discuss possible areas of future research.

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