The Influence of Manipulatives on Students' Understanding of Advanced Algebra Concepts

By

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To my girls, Karis and Lauren, who inspired me to take this educational journey. You both enrich my life and I love you with all my heart. I hope during these three years of study you have seen me live out my conviction that anyone is capable of completing a difficult task with the grace of God. We must never give up because 2 Corinthians 12:9 says, "… 'My grace is sufficient for you, for my power is made perfect in weakness. …'".

Acknowledgments

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Abstract

Practice and research show that many Intermediate Algebra students have problems with comprehending sets, inequalities, graphs, and 3 dimensional concepts. The purpose of this thesis is to examine if using manipulatives is helpful in learning these concepts. Extensive use of manipulatives helps, usually in preschool – 5th grade. Manipulatives are objects that are touched, moved, or "manipulated" in some way and are the basis for independent explorations. Analysis of data collected in this pilot study shows evidence of improved students' performance on sets, inequalities, graphing, and understanding of 3 dimensions after students had the opportunity to "experience" these ideas via activities involving manipulatives. Additionally, our surveys show that students' attitude changed toward mathematics and, for some, this increased their desire to take higher level courses.

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Chapter 1 - Introduction

Preschool children primarily learn using manipulatives. According to Jean Piaget's cognitive-developmental theory, children use manipulatives to learn about and understand their world (Berk, 2005, p. 20). But what about adults? According to a study done on the development of formal operations in logical and moral judgment, merely 30% of adults fully attain conceptual thinking while 15% think only in tangible ways (Kuhn, Langer, Kohlberg & Haan, 1977). So how can we bridge the gap between the tangible and the theoretical? Manipulatives may possibly be the path that unlocks the door to understanding the world, limited only by our imagination.

Mathematics is defined by "the study of quantity, structure, space, and change" (Wikipedia, 2011). Intermediate Algebra is a part of mathematics that integrates these well. It moves from the concrete to the abstract in a beautiful way. Unfortunately, sixty-percent of nationwide college-bound freshman are not proficient in Algebra (ACT, 2005). This means that these students need to complete remedial courses via Elementary and/or Intermediate Algebra classes in order to be admitted into the university system. So what if we incorporate manipulatives with the learning of Intermediate Algebra concepts, specifically sets, inequalities, graphs, and 3 dimensional thought? Would that improve the learning curve? Six manipulative activities were designed to cover these concepts in hopes of seeing an improvement in performance in these areas. With the analysis complete, the outcome does show that these manipulatives improve students' performance in these specified areas. We used Excel for statistical analysis and an example of raw data are included in Appendix U.

Chapter 2 - Description of Research

2.1 Location of Study

This pilot study took place at California State University at Channel Islands (CSUCI), located in the city of Camarillo, CA, 53 miles North of Los Angeles and 43 miles South of Santa Barbara.

2.2 Participants of Study

The students who participated in this study were enrolled during the Fall 2010 semester in the course Math 95, Intermediate Algebra, at CSUCI. Math 95 is a credit/no credit five unit class, consisting of four hours of lecture and one hour of lab. Math 95 was offered in five sections, 01 – 05. The students in this study were in the second section, 02. This section met Monday and Wednesday, 10:00 AM – 10:50 AM and Friday, 10:00 AM – 11:50 AM. There were twenty-nine students enrolled but only twenty-three students participated in the study. Math 95 is a required math course for students who did not pass the ELM, i.e. are not ready for college level math courses. This course is a review of Geometry and Intermediate Algebra. The subject matter includes functions and graphs, systems of linear equations, inequalities, exponents and radicals, quadratic functions, and exponential and logarithmic functions.

2.3 Duration of Study

The six activities, two pre-tests, two questionnaires, one mid-test, and one post-test were conducted over twelve days throughout the semester. The first day of the semester was August 30 and the last day was December 13. The days this study was carried were August 30, September 1, 3, 10, 17, 24, October 15, 29, November 5, 8, 12, and December 3.

2.4 Outline of Study Method

A total of six activities were presented on seven different days. Before the commencement of the activities, two pretests were administered on two different days. Pre-Test - Part 1 (Appendix A) consisted of graphing inequalities and points on a number line, graphing inequalities in 2 dimensions, having to describe an equation or inequality in 2 dimensions, tangent lines, the intersection and union of sets, and the distinction of discrete and continuous sets. On the same day that the Pre-Test – Part 1 was completed, Student Info Questions A (Appendix Q) was also filled out. Pretest - Part 2 (Appendix B) contained inequalities with variables rather than numbers, finding equations to system of inequalities, graphing system of inequalities, and knowing equations in 2 and 3 dimensions. Also included in part 2 was graphing a line in 2 dimensions and overlaying it with parabolas that initially had variables in the equations which were then reconstructed by the student who decided what number to use as the variable to allow the line and parabolas to intersect. Pre-Test – Part 1 was the heart of activities 1 – 3 and for Pre-Test - Part 2, activities 4 – 6.

After the pre-tests were given, the students were ready to begin the activities. Before each activity, a questionnaire describing the students' confidence level before the activity was filled out. After each activity, a 5 - 10 minute post activity test was given to the students, worth up to 10 extra credit points for correct answers. This helped reinforce what the students had learned. Also, the second portion of the questionnaire was then filled out to describe the students' confidence level after each activity.

The study was conducted during regularly scheduled class sessions that included regular lectures, testing, and homework assignments. Each activity lasted about 1 hour except for activity 6 which took about 2 hours and 30 minutes in total.

Activities 1 – 3 were given within the first three weeks of class. Activity 1 covered discrete sets using Venn diagrams. Activity 2 covered continuous sets and inequalities using bamboo skewers, fake money, and the concept of time. Activity 3 focused on continuous sets, graphing points, lines, and inequalities in 0, 1, 2, and 3 dimensions using playdough.

A week after activity 3, a 30 minute mid-test was given to the students. This mid-test was exactly the same test as pretest part 1, except the two questions related to tangent lines were removed because it was decided not to include that subject in this study.

Three weeks later, activities 4 – 6 began. Activity 4 covered graphing inequalities in 2 dimensions using colored transparencies. Two weeks later, activity 5 began which covered graphing, inequalities, and equations in 0, 1, 2, and 3 dimensions using

a game called Graph Blast Bingo. Activity 6 was split into two days. It started a week after activity 5 and was completed on the following class. Activity 6 covered graphing, inequalities, and equations in 3 dimensions using aquariums and transparencies. The first day was used to construct the aquariums and the second day was used to carry out the activity.

Two classes after the completion of activity 6, a second questionnaire, Student Info Questions B (Appendix R), was given to the students to fill out. Three weeks after that, the Post-Test was given to the students. The Post-Test was the same test as Pre-Test - Part 2. This concluded the collection of the data.

Note the visual overview below.

Pre-Test - Part 1
Student Info Questions A

Pre-Test - Part 2

Confidence Level Questionnaire (Pre-Activity 1)
Activity 1
    Post-Activity Test
Confidence Level Questionnaire (Post-Activity 1)

Confidence Level Questionnaire (Pre-Activity 2)
Activity 2
    Post-Activity Test
Confidence Level Questionnaire (Post-Activity 2)

Confidence Level Questionnaire (Pre-Activity 3)
Activity 3
    Post-Activity Test
Confidence Level Questionnaire (Post-Activity 3)

Mid-Test

Confidence Level Questionnaire (Pre-Activity 4)
Activity 4
Post-Activity Test
Confidence Level Questionnaire (Post-Activity 4)

Confidence Level Questionnaire (Pre-Activity 5)
Activity 5
Post-Activity Test
Confidence Level Questionnaire (Post-Activity 5)

Confidence Level Questionnaire (Pre-Activity 6)
Activity 6
Post-Activity Test
Confidence Level Questionnaire (Post-Activity 6)

Student Info Questions B
Post-Test

The Post-Activity tests were mini-tests that were considered part of each activity. They were given to the students so they could apply what they just learned and to help solidify their understanding.

2.5 Detailed Description of the Experiment

Monday, August 30, 2010

This day was the first day of class. Students were given approximately 30 minutes to take Pre-Test - Part 1. This pretest covered more applied mathematical concepts and 1 and 2 dimensions. After this, they were given Student Info Questions A to fill out.

Wednesday, September 1, 2010

Students were given approximately 30 minutes to take Pre-Test - Part 2. This pretest covered abstract mathematical concepts and 2 and 3 dimensions.

Friday, September 3, 2010

Activity 1 was administered after a regular lecture. This lecture was not part of the research. Just before the activity started, the first half of the Activity 1 Questionnaire (Appendix E) was filled out. Activity 1 started around 10:30 AM and it focused on Venn diagrams and discrete sets. To begin, two different sets were given to each student to examine with their hands and decide what properties were important for their analysis. Later, a third set was handed out with qualities that were not the same as the other sets which required the students to reason how they could incorporate this third set into their Venn diagrams. They looked at sets in two different ways, separating elements by color and counting cardinalities of subsets and experienced building unions, intersections, using subsets, and dealing with a special case for disjoint sets.

When the activity was over, the students took the Post Activity 1 (Appendix F) and completed the Activity 1 Questionnaire.

Friday, September 10, 2010

Activity 2 exposed the students to continuous sets and inequalities. We concentrated on two explorations titled: Part 1: Inequalities on Bamboo Skewers and Part 2: Continuous Sets Using Money and Time that I have designed. Just before the activity began, the first half of the Activity 2 Questionnaire (Appendix G) was filled out.

For part 1, the students used highlighters to color a bamboo skewer to shade two different lengths. Each length was represented by a different color so that the overlap would turn into a new color. For instance, if yellow was used for the first

inequality and blue was used for the second inequality, the intersection would be green. The students could clearly see what was the intersection and union of the two lengths.

We explained what it means when real numbers form a continuous set, and worked with subsets. Also, we explained how a continuous set differs from a discrete set.

For part 2, each student received fake money to count and claim as their current balance in their pretend bank accounts. Deposits and withdrawals were made. After each change in their accounts, the students wrote down an inequality that represented the amount of money they may spend given there is no minimum amount required in their bank account and another inequality that requires a minimum of $25 in their account. We discussed the negative balances and addition of debts. We also discussed large sums of money and an infinite amount of money, large debts and an infinite amount of money owned. Each student concluded that time is represented by real numbers, hence is continuous.

Now we focused on intersections and unions. The students pretended they were getting married and their new spouse shared his/her checking account with them. They looked at both accounts and evaluated different scenarios on how they could spend the money. They used inequalities to describe their situations.

When the activity was over, the students took the Post Activity 2 (Appendix H) and completed the Activity 2 Questionnaire.

Friday, September 17, 2010

Activity 3 used playdough to give the students a better understand of graphing in 0, 1, 2, and 3 dimensions. This activity was separated into three parts: 0 and 1 dimensions, 2 dimension, and 3 dimension. Just before activity started, the first half of the Activity 3 Questionnaire (Appendix I) was filled out.

Part 1

We discussed measurements in 0 and 1 dimensions. The students were told to use one skewer as the number line and playdough to create an example of a discrete set (0 dimension) by placing little balls on the skewer and a continuous set (1 dimension) by placing a "worm" shape of playdough on the skewer. With this information, they were asked to describe these points in set notation, using inequalities or interval notation, for example (1, 2), and graph them on the line.

Part 2

a) We discussed the coordinate system in 2 dimensions using a piece of paper with a Cartesian coordinate system printed on one side. Then as before, the students used playdough to make points to represent a discrete set and a "worm" shape to look like a line or parabola or some other curve that they knew and then represent these two sets with set notation, inequalities, and/or interval notation.

b) The students took the playdough to form paper-like shape to "shade" the area under the graph and above the x-axis. They figured out inequalities describing the set.

Part 3

a) We discussed the coordinate system in 3 dimensions using three skewers and twist ties. Students constructed a 3-axis coordinate system symbolizing 3 dimensional space. Then, using playdough, they made a discrete set of points with its coordinates (most of the points were on the axes) and a large sphere in 3 dimensions with its equation. They spent time trying to figure out equations for the spheres.

When the activity was over, the students took the Post Activity 3 (Appendix J) and completed the Activity 3 Questionnaire.

Friday, September 24, 2010

A 30 minute Mid-Test (Appendix C) was administered. It was the exact test as the Pre-Test – Part 1 with one exception. The Pre-Test – Part 1 had a question about finding a tangent line that was taken out of the Mid-Test as we did not cover the topic. The first three activities focused on the material in the Pre-Test – Part 1 and the Mid-Test.

Friday, October 15, 2010

Activity 4, done right in the middle of the semester, was focused on the students' understanding of inequalities. Just before the activity began, the first half of the Activity 4 Questionnaire (Appendix K) was filled out.

Each student was given a worksheet with four graphing questions on it. Each question included a grid for graphing.

Students started with graphing boundaries of these inequalities as lines first. The students matched edges of transparencies to represent the shaded area solving each inequality. The overlapping transparencies gave different darker colors describing a common solution. This was repeated with questions 2 – 4. For problem 4, the students used a cut out a colored transparent parabola that they overlapped with areas given by lines.

When the activity was over, the students took the Post Activity 4 (Appendix L) and completed the Activity 4 Questionnaire.

Friday, October 29, 2010

This activity involved a bingo game focused on equations and graphing in 0, 1, 2, and 3 dimensions. Just before activity 5 began, the first half of the Activity 5 Questionnaire (Appendix M) was filled out.

There was a quick graphing review before the bingo game.

The students each picked one bingo card (Appendix S) which had four boxes on them. In each box there was an equation. The equation was one of the following: a point, a line or inequality on the number line; a line, parabola, absolute value equation, a polynomial, a circle, or inequality in 2 dimensions; a plane or sphere in 3 dimensions.

On the computer screen in front of the class, one graph at a time was shown without its equation (Appendix T). The students had to connect the graph with its

equation. When they did, they put a marker on the box. The person who filled up their card first won the game.

When the activity was over, the students took the Post Activity 5 (Appendix N) and completed the Activity 5 Questionnaire.

Friday, November 5, 2010

Activity 6 was concerned with graphing in 3 dimensions. The first half of the Activity 6 Questionnaire (Appendix O) was filled out before the activity began.

On the first day of the activity the students assembled their aquariums.

Each student was given a cubic, plastic aquarium, rocks of their choice, shells, starfish, masking tape (to label aquarium with name), and gelatin. They put the items in their own aquariums and added gelatin (dissolved in water) filling their aquariums approximately 1 inch from the top.

Monday, November 8, 2010

On the second day of Activity 6 the finished aquariums were used to delve into 3 dimensions beginning with individual assessment, followed by a two-player game, and lastly with the construction of equations in 3 dimensions.

Each student was given a small colored plastic fish and inserted it into the gelatin with a skewer. To describe the location of the fish we introduced the grid-lined transparencies which were taped to the sides of the aquarium, which represented

the x-axis, y-axis, and z-axis and grid points. Students visually decided what coordinates best represented the location of their fish.

Students were divided into groups of two to play the game. To locate the other's fish, each opponent asked questions in the form of an inequality, such as "Is your x-coordinate greater than or equal to 2?". The winner of each team not only located the opponent's fish first, but had to answer the location as a point in 3 dimensions, such as (3, 5, 1).

We discussed parametrization of a line and equations for a plane and a sphere in 3 dimensions. We practiced translations of and changes to the equations.

When the activity was over, the students took the Post Activity 6 (Appendix P) and completed the Activity 6 Questionnaire.

Friday, November 12, 2010

The second midterm was given which was unrelated to the research. After the students finished their exam, they were given Student Info Questions B to fill out.

Friday, December 3, 2010

Four class days before the final, the students took the 30 minute Post-Test (Appendix D). This Post-Test was the same exact test as the Pre-Test – Part 2. The last 3 activities focused on the material in the Pre-Test – Part 2 and the Post-Test.

Chapter 3 - Method

In this study we covered sets, inequalities, graphs, and 3 dimensional concepts in algebraic context. The method used involved manipulatives which allow students to touch, move, and manipulate real objects for independent exploration and learning. We designed and implemented six activities. Activities 1 – 5 were completed during one class meeting each. Activity 6 was accomplished during two class periods.

3.1 Pre-Tests, Mid-Test, Post-Test, and Questionnaires

Two pre-tests, a mid-test, a post-test, and questionnaires were set to monitor the progress and thoughts of the students as well as to get some background information.

3.1.1 Pre-Tests

Pre-Test – Part 1 (Appendix A) covered graphing inequalities and points on a number line, graphing inequalities in 2 dimensions, having to describe an equation or inequality in 2 dimensions, tangent lines, the intersection and union of sets, and the distinction of discrete and continuous sets. Pre-Test – Part 2 (Appendix B) covered inequalities with coefficients given as parameters, finding equations to system of inequalities, graphing system of inequalities, and equations in 2 and 3

dimensions. We also included here graphing a line on a plane and overlaying it with parabolas that initially had undefined parameters as coefficients. Pre-Test – Part 1 was the heart of activities 1 – 3 and for Pre-Test - Part 2, activities 4 – 6. These pretests were given on the first and second day of class.

3.1.2 Mid-Test and Post-Test

The Mid-Test (Appendix C) was given after activity 3 and a Post-Test (Appendix D) was given after the last activity, activity 6. The Mid-Test was almost identical to Pre-Test – Part 1. Two questions related to tangent lines were removed because that subject was not included in this study. The Post-Test was identical to the Pre-Test – Part 2.

3.1.3 Questionnaires

Two general questionnaires, Student Info Questions A (Appendix Q) and Students Info Questions B (Appendix R), provided background data on each student. Questionnaire A was given on the first day of class and questionnaire B was given after all the activities were completed. The confidence level questionnaires (Activity 1 Questionnaire, etc.) for each activity had two parts. The first page represented the students' evaluation of their confidence level on a subject before the activity began and the second page described the students' evaluation of their confidence level on the same subject after the activity was finished. All of these questionnaires can be found in the appendices.

3.2 Activity 1: Venn Diagrams – Discrete Sets

Activity 1 focused on Venn diagrams and discrete sets and was administered after a regular lecture. Just before the activity started, the first half of the Activity 1 Questionnaire (Appendix E) was filled out.

Activity 1: Venn Diagrams; Discrete Sets

Supplies:
Several types of colored items: small deflated balloons, paper clips, pencil top erasers, rubber bands, birthday candles, clear plastic crystal gems

Each student was given a large piece of paper and two sets of items from the list above. Each group had items in three colors and only one color repeated in both groups (that will be used as a common property – an intersection). For example, a student received: 1 pink, 2 green, 3 yellow balloons; and 2 pink, 3 orange, and 5 blue paper clips. In this case the common color was pink.

The students drew two intersecting circles, Venn diagrams, on the paper showing an intersection of two sets, labeling them A or B. Using their items, they organized the two sets A & B in such a way that items in the common color were in both sets, and made a list of items in each set (numbering some items to distinguish them). Then the students placed all items in the circles in such a way that the common part included items of the same color. After everyone completed this step, students described their situation using set notation for various sets as follows:

Step 1: Elements of each set A and B were also represented using only colors to form new sets A_{colors} and B_{colors}. Students performed various set operations on these sets.

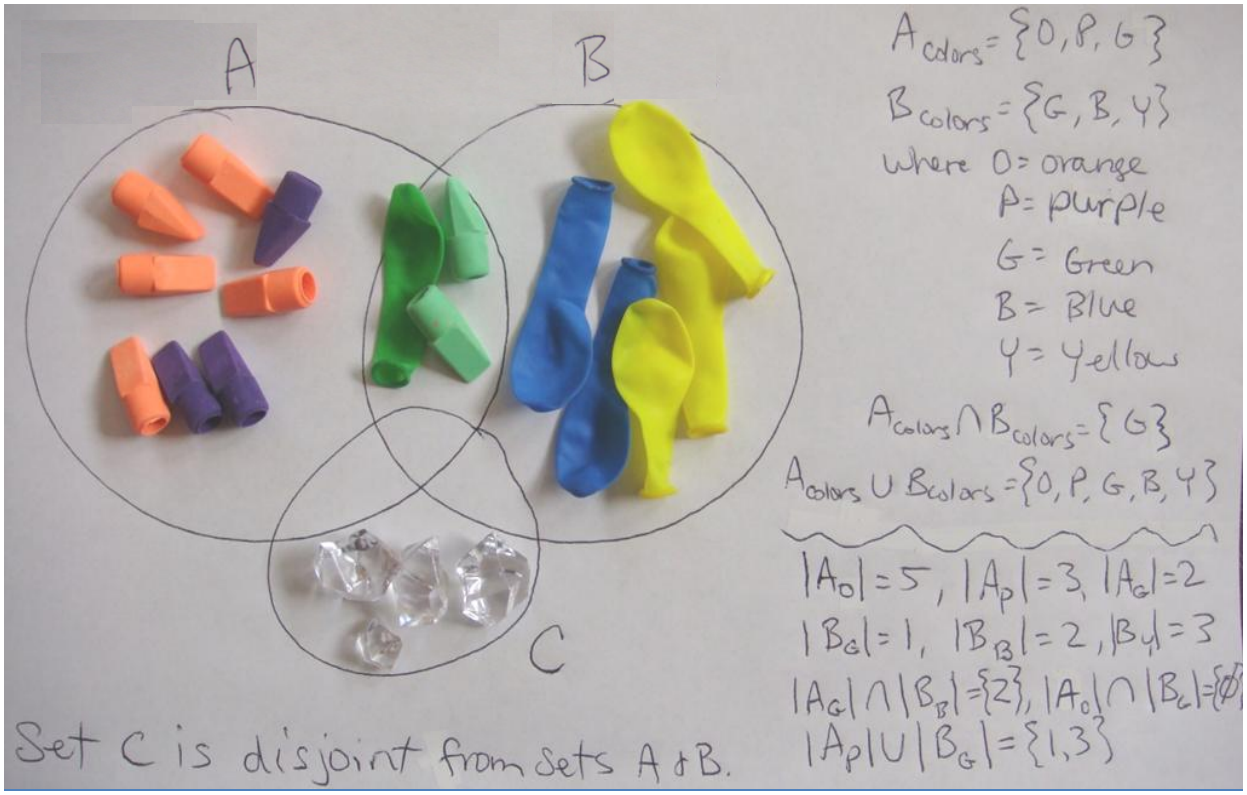
Step 2: Students calculated cardinality of various sets, including subsets of each color within the sets A and B.

Example: Using the items above, A = {all balloons and all pink items} and B = {all clips and all pink items}. $A \cup B$ = {all items}, $A \cap B$ = {all pink items}. A_{colors} = {P, G, Y} and B_{colors} = {P, O, B}. Then $A_{colors} \cap B_{colors}$ = {P} and $A_{colors} \cup B_{colors}$ = {P, G, Y, O, B}, where P = pink, O = orange, B = blue, G = green, and Y = yellow.

Step 2 in our example, $|A_P| = 3$, $|A_G| = 2$, $|A_Y| = 3$, $|B_P| = 3$, $|B_O| = 3$, and $|B_B| = 5$.

We looked at different properties defining intersections and unions of A & B and their subsets.

Step 3: Each student was given a few of the clear plastic crystal gems and told that this is set C. We wanted them to incorporate this set with their picture in a meaningful way. Students realized that the properties we considered so far made set C disjoint from A and B (there were no clear colors in any of the sets). We worked out an appropriate Venn diagram, two intersecting circles, and one disjoint from the previous ones. We again, played with the intersections and unions but with sets A, B, and C, and their subsets. We verbally described each situation.



In this activity, the students were able to examine sets with their hands and decide what properties are important for their analysis. They used Venn diagrams to look at sets in two different ways, separating elements by color and counting cardinalities of subsets. They also experienced building unions, intersections, using subsets, and dealing with disjoint sets.

When the activity was over, the students took the Post Activity 1 (Appendix F) and completed the Activity 1 Questionnaire.

3.3 Activity 2: Bamboo Skewers – Continuous Sets

Activity 2 exposed the students to continuous sets and inequalities. We designed two explorations titled: "Inequalities on Bamboo Skewers" and "Continuous Sets Using Money and Time". Just before the activity began, the first half of the Activity 2 Questionnaire (Appendix G) was filled out.

Activity 2: Bamboo Skewers and Money & Time; Continuous Sets and Inequalities

Part 1: Inequalities on Bamboo Skewers

Supplies:
Ruler (12 inches) - representing the number line
9-inch Bamboo Skewer (tip removed) – representing sets
Two different colored highlighters

Each student was given a ruler, one bamboo skewer, two highlighters, and two inequalities in one variable. The students placed the skewer so it started out next to the ruler, which acted as a number line, and marked the solution to the first inequality on the skewer. This measurement was colored in one color.

The second inequality was found and was colored in the second color. The solution sets of the inequalities overlapped one another on the number line, for example, $3 \leq x \leq 6$ and $5 \leq x \leq 8$. After they colored their skewers, the students were asked to find the intersection and union of the two sets representing solutions to their inequalities. The union was the entire colored area and the intersection was the new color created by the overlapping of two different colors.

Students used inequalities to write down unions and intersections of sets. They answered questions such as, "Would 3.7265 be a part of your solution set?" We explained what continuous sets are and interpreted them on the number line. Also, we explained how a continuous set differs from a discrete set.

Part 2: Continuous Sets Using Money and Time

Supplies:

One baggie of fake money to represent the current balance in their account

One baggie of fake money to represent a deposit into their account

Two paper debit transactions (labeled with date and dollar amount)

Each student counted their current balance and then wrote down the balance as an item in set B (for balance). Each student then counted their deposit and wrote down the amount as an item in set D (for deposits). Sets representing money involved in bank transactions were labeled as W_S. Students marked their balances on a number line, then added or subtracted the amounts and labeled the new balances on the graph. At some point they applied a withdrawal which would bring the balance below zero which meant they had to perform calculations on negative numbers. After each change in their accounts, the students wrote down an inequality that represented the amount of money they may spend given there is no minimum amount required in their bank account and another inequality that requires a minimum of $25 in their account. We discussed the negative balances, addition of debts, large sums of money, and an infinite amount of money, large debts and an infinite amount of money owned. The students decided the set of all theoretically possible withdrawals and deposits (and our total amount) can be

represented by real numbers, hence is continuous (as banks can count even smallest fractions of a penny). They made a conclusion that time is also a continuous set.

To study intersections and unions, students pretended they were getting married considering opening a shared checking account with their spouse. The amount in his/her account is equal to $24.32 and he/she has unlimited overdraft with no minimum needed in the account. Students figured out the maximum amount they could write a check for assuming they did not want to combine the accounts, and the maximum amount that they could write a check for if they combine the accounts. Then they were asked to explain what would happen if they wanted to spend more than the maximum. Then they were asked to think about what would happen to the maximum if they moved some money between the two accounts. They used inequalities to describe their situations.

Post Activity 2 (Appendix H) and the Activity 2 Questionnaire were administered at the end of this activity.

3.4 Activity 3: Playdough – Graphing in 0 – 3 Dimensions

Activity 3 used playdough to give the students a better understand of graphing in 0, 1, 2, and 3 dimensions. Just before activity started, the first half of the Activity 3 Questionnaire (Appendix I) was filled out.

Activity 3: Playdough; Continuous Sets, Graphing, 0, 1, 2, & 3 dimensions, and Inequalities

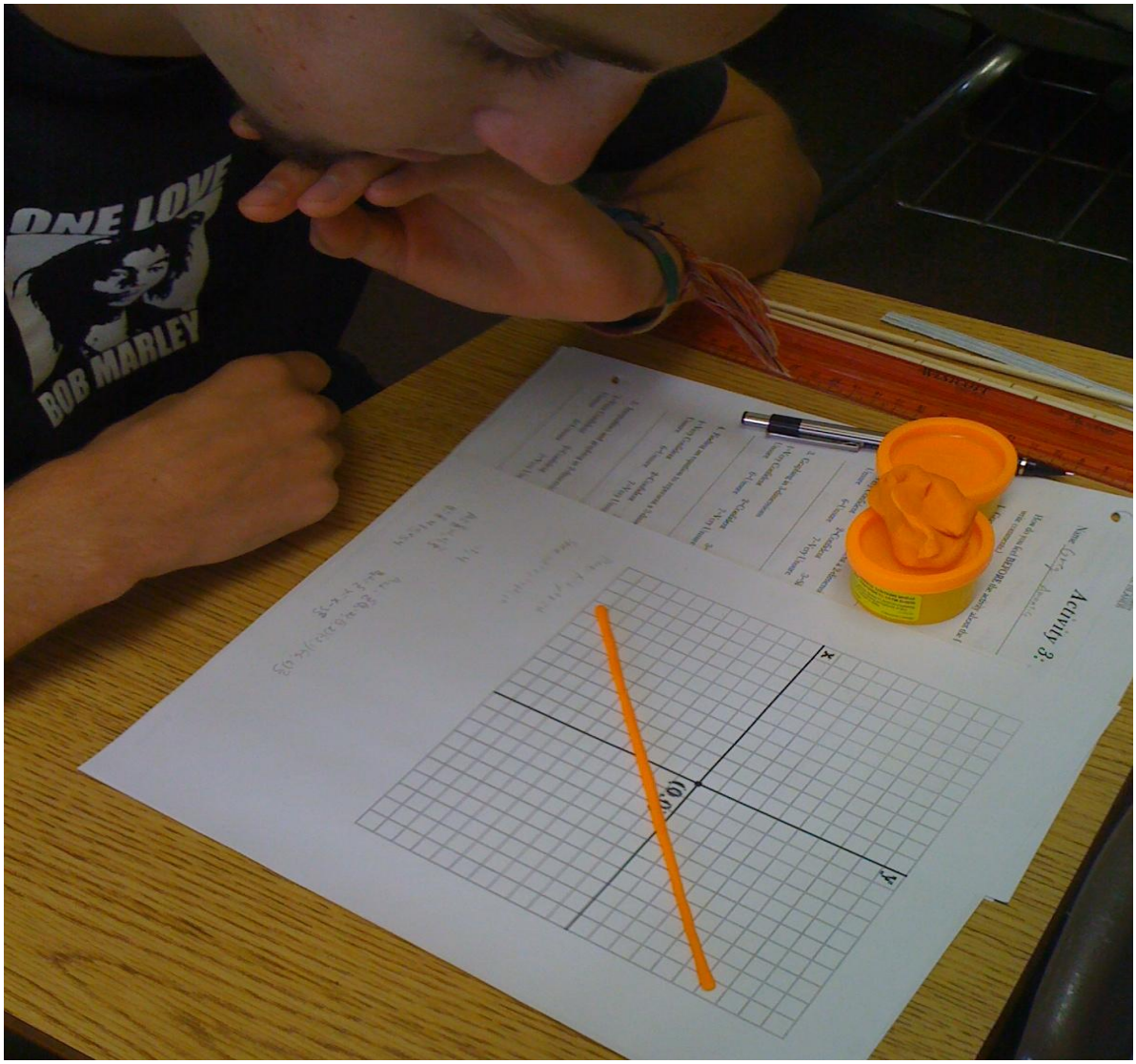
Supplies:
Three wooden skewers
A piece of paper with a Cartesian plane printed in the middle
A ruler
Two or three twist ties
Two small containers of playdough

The students were given three skewers and were told to line them up. On each skewer, they marked the midpoint with a dark line and marked out 1 inch measurements on either side of the midpoint.

Part 1

Students used one skewer as the number line and randomly placed little balls of playdough on this number line. The balls represented a discrete set where each

ball is in 0 dimensions. Using the ruler, the students decided what real number value their points represent. They presented these points as a set using set notation. Next students placed (straight) "worm" shape (representing a continuous set in 1 dimension) on the skewer and described this set using inequalities or interval notation (for example (1, 2)).



Part 2

a) For 2 dimensions, the paper with a Cartesian coordinate system printed on one side, was used by the students to place several playdough points on the plane. Students wrote down the coordinates of each point. Then students placed a "thin" continuous set (a "worm" shape) to represent a line or parabola or some other curve of interest. Students figured out equations of their "worms" (in x and y coordinates).

b) The students used flattened pieces of the playdough to "shade" the area under the graph and above the x-axis. They figured out inequalities describing the sets.

Part 3

We discussed the Cartesian coordinate system in 3 dimensions using three skewers and twist ties. Students constructed a 3-axis coordinate system symbolizing 3 dimensional space. Then, using playdough, they made a discrete set of points in 3 dimensions and gave coordinates (most of the points were on the axes). Then students made a sphere and placed it within the coordinate system (usually off the origin). They spent time trying to figure out equations for the spheres. We worked out the general formula for a sphere (with center (x_0, y_0, z_0) and radius r in (x, y, z) coordinates) as $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$. Students worked out the equation in their cases, and checked if points on the sphere satisfied their equation.

When the activity was over, the students took the Post Activity 3 (Appendix J) and completed the Activity 3 Questionnaire.

3.5 Activity 4: Transparencies – Graphing Inequalities in 2 Dimensions

Activity 4 was focused on the students' understanding of inequalities. Just before the activity began, the first half of the Activity 4 Questionnaire (Appendix K) was filled out.

Activity 4: Transparencies; Graphing with Inequalities in 2 dimensions

Supplies:
Worksheet with graphing paper
Colored clear transparencies (in various colors per sheet: blue, yellow, red, …)
Scissors

Students were given a worksheet with four graphing questions on it. Each question included a grid for graphing.

EXAMPLE:

Directions: Graph each set of inequalities onto the corresponding Cartesian plane. Use transparencies to shade desired areas.

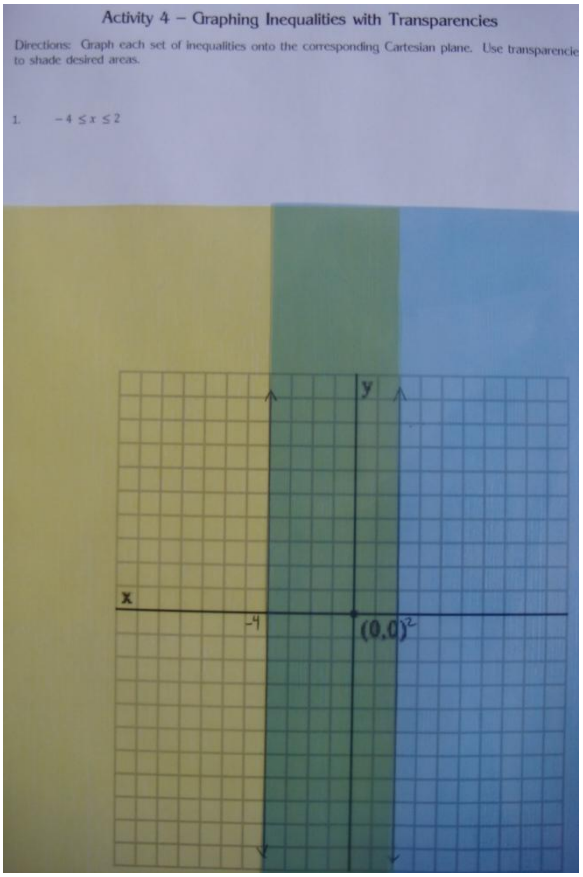
1. $-4 \leq x \leq 2$

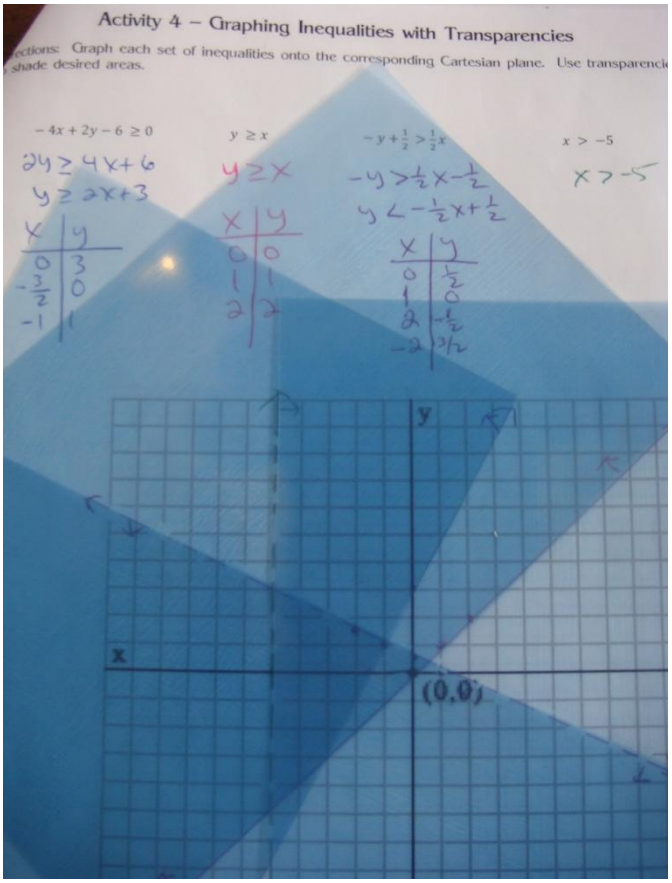
2. $-8x + 4y - 4 \geq 0$ $\quad y - 1 < -\frac{1}{2}x$ $\quad -y < x + 2$ $\quad x \geq -4$

3. $-4x + 2y - 6 \geq 0$ $\quad y \geq x$ $\quad -y + \frac{1}{2} > \frac{1}{2}x$ $\quad x > -5$

4. $y \geq x^2 - 2$ $\quad y < x$ $\quad y \geq 0$

Students started with graphing boundaries of these inequalities as lines first. They used edges of transparencies as straight lines to represent the shaded area solving each inequality. The overlapping transparencies gave different darker colors describing a common solution. For problem 4, the students used a cut out a colored transparent parabola that they overlapped with areas given by lines.





When the activity was over, the students took the Post Activity 4 (Appendix L) and completed the Activity 4 Questionnaire.

3.6 Activity 5: Graph Blast Bingo – Equations in 0 - 3 Dimensions

This activity involved a bingo game focused on graphing in 0, 1, 2, and 3 dimensions. Just before activity 5 began, the first half of the Activity 5 Questionnaire (Appendix M) was filled out.

Activity 5: Graph Blast Bingo; Graphing, Inequalities, and Equations in 0, 1, 2, & 3 Dimensions

Supplies:
Bingo game (graphs without equations)
Equation bingo cards

There was a quick graphing review before the bingo game.

The students each picked one bingo card (Appendix S) which had four boxes on them. In each box there was an equation of either a point, a line, a parabola, absolute value equation, a polynomial, a circle, a plane or sphere in 3 dimensions, or an inequality. Different colors were used to represent each of these graphs as shown below.

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

On the computer screen in front of the class, one graph at a time was shown without its equation (Appendix T). There was a small area on the screen that had the color that represented that graph. This coloring system helped the students organize their thoughts and connect the graph with the equation. When the students recognized that they had the equation to the graph on the screen, they put a marker on the box. The person who filled up their card first won the game.

When the activity was over, the students took the Post Activity 5 (Appendix N) and completed the Activity 5 Questionnaire.

3.7 Activity 6: Aquarium – 3 Dimensions

Activity 6 was concerned with graphing in 3 dimensions. The first half of the Activity 6 Questionnaire (Appendix O) was filled out before the activity began.

Activity 6: 3 Dimensional Fish Bowl; Inequalities, Graphing, and Equations in 3 Dimensions

Supplies:
Clear, rectangular, plastic aquarium with lid
Small rocks (bags of different colors)
Shells and starfish
Four small packets of gelatin
Hot and cold water
Masking tape
Grid-lined transparency
Scissors
Scotch tape

Mini fish-shaped glass beads
Wooden skewers
Solid-colored transparencies

Day 1: Assemble Aquariums

Each student was given a cubic aquarium, rocks, shells, starfish, masking tape (to label aquarium with name), and gelatin. They arranged the items in their own aquariums and mixed the gelatin in a separate container. They filled their aquariums with the gelatin and added cold water to fill approximately 1 inch from the top.

When Activity 6 was continued during the following class period, the finished aquariums were used to delve into 3 dimensions beginning with individual assessment, followed by a two-player game, and lastly with the construction of equations in 3 dimensions.

Day 2 – Part 1: Fish Location

Each student was given a small colored plastic fish and inserted it into the gelatin with a skewer. To describe the location of the fish we introduced the grid-lined transparencies which were taped to the sides of the aquarium, which represented the x, y, and z-planes and grid points. Students visually decided what coordinates best represented the location of their fish.

Day 2 – Part 2: Fish Location Game

Students were divided into pairs to play the game. Neither was able to see the aquarium of the other but could only ask questions in the form of an inequality, such as, "Is your x-coordinate greater than or equal to 2?". It took on average 10 – 15 questions to locate the fish of the opponent. The winner of each team not only located the opponent's fish first, but had to answer the location as a point in 3 dimensions, such as (3, 5, 1).

Day 2 – Part 3: Plane Inequality and Sphere Equation

We discussed parametrization of a line and equations for a plane and a sphere in 3 dimensions. We practiced translations of and changes to the equations.

Each student inserted a piece of solid-colored transparency and half of a clear, plastic, empty ball into their aquarium. The transparency was used as a plane and the ball as a hemisphere. Each student figured out the equation for the plane and

the equation of the sphere. Then students worked on inequalities describing the location of the "sea life" inside the hemisphere.

When the activity was over, the students took the Post Activity 6 (Appendix P) and completed the Activity 6 Questionnaire.

Chapter 4 - Results

To support our research we compared the mean of the differences for our data sets. Specifically, we tested the following hypothesis:

H_0: $\mu_d \leq 0$. The mean of the Post-Test/Pre-Test differences is less than or equal to 0.

H_a: $\mu_d > 0$. The mean of the Post-Test/Pre-Test differences is greater than 0.

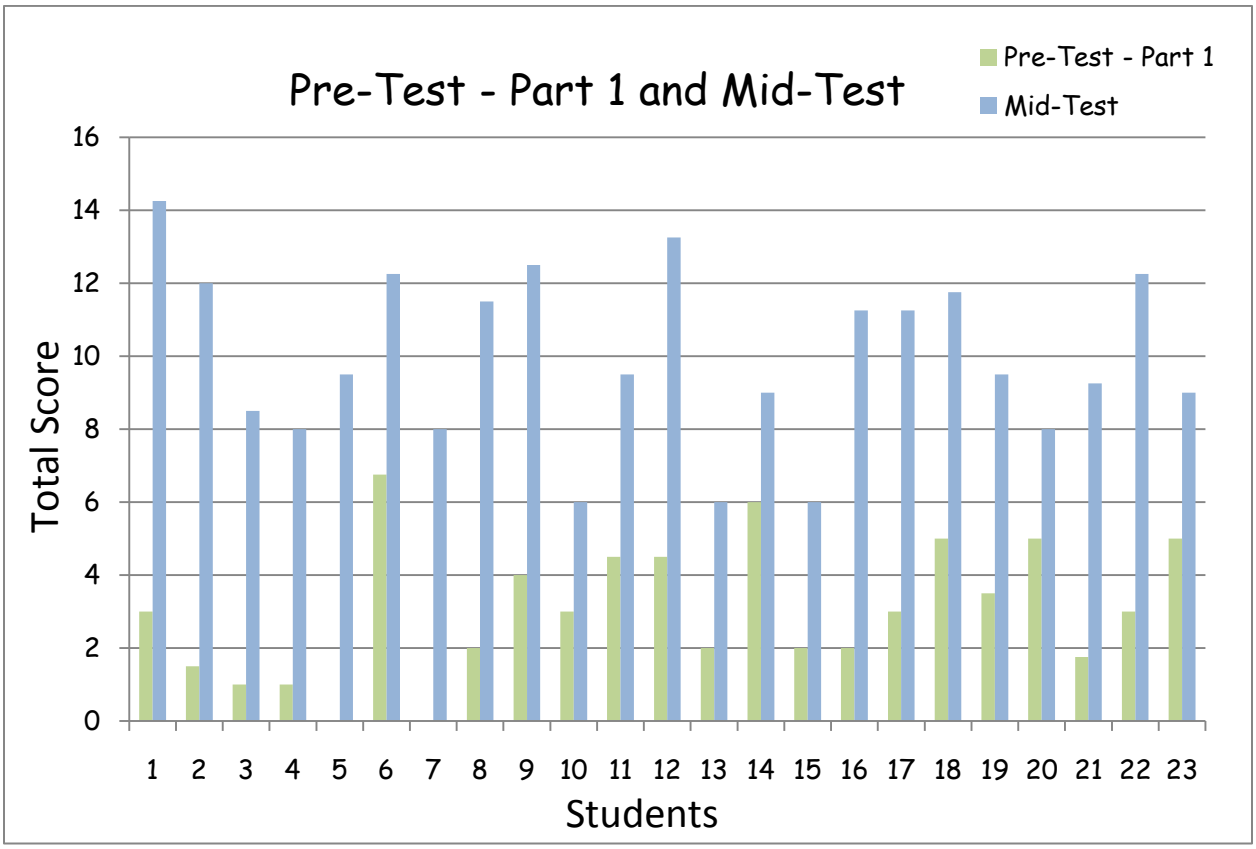
Pre-test in this context means testing results before and activity and the mid-test and the post-test will represent the results after the activity. These pre/post-test scores were also compared by subject matter. Since the population variance is unknown and the sample size was relatively small (twenty-three), a t-test was performed on the data. Also, right-sided, one-tailed p-values were looked at with a confidence level of 95% which translates to an α-value of 5% or 0.05. For each statistical analysis, we compared the p-value with the above α-value to decide if the difference is significant to accept or reject the hypothesis.

4.1 Results from Pre-Test – Part 1 and Mid-Test

The data from Pre-Test – Part 1 (Appendix A) and the Mid-Test (Appendix C) were compared and analyzed. These two tests examined the students' understanding of graphing inequalities and points on a number line, graphing inequalities in 2 dimensions, having to describe an equation or inequality in 2 dimensions, the intersection and union of sets, and the distinction of discrete and continuous sets. These concepts were learned via activities 1 – 3. The Mid-Test has identical to

Pre-Test – Part 1 except for two questions that were removed from Pre-Test – Part 1 because that subject was not included in this study (and related data was removed from pre-test scores). Pre-Test – Part 1 data is colored light blue and Mid-Test data is colored light green in the graphs.

Pre-Test – Part 1 and Mid-Test were out of 19 points. One student had the lowest pre-test score of a zero. The highest score was 6.75. The mean was 3.02 and the standard deviation was 1.84. On the mid-test, three students had the lowest score of a 6 and the highest score was 14.25. The mean was 9.93 and the standard deviation was 2.36.

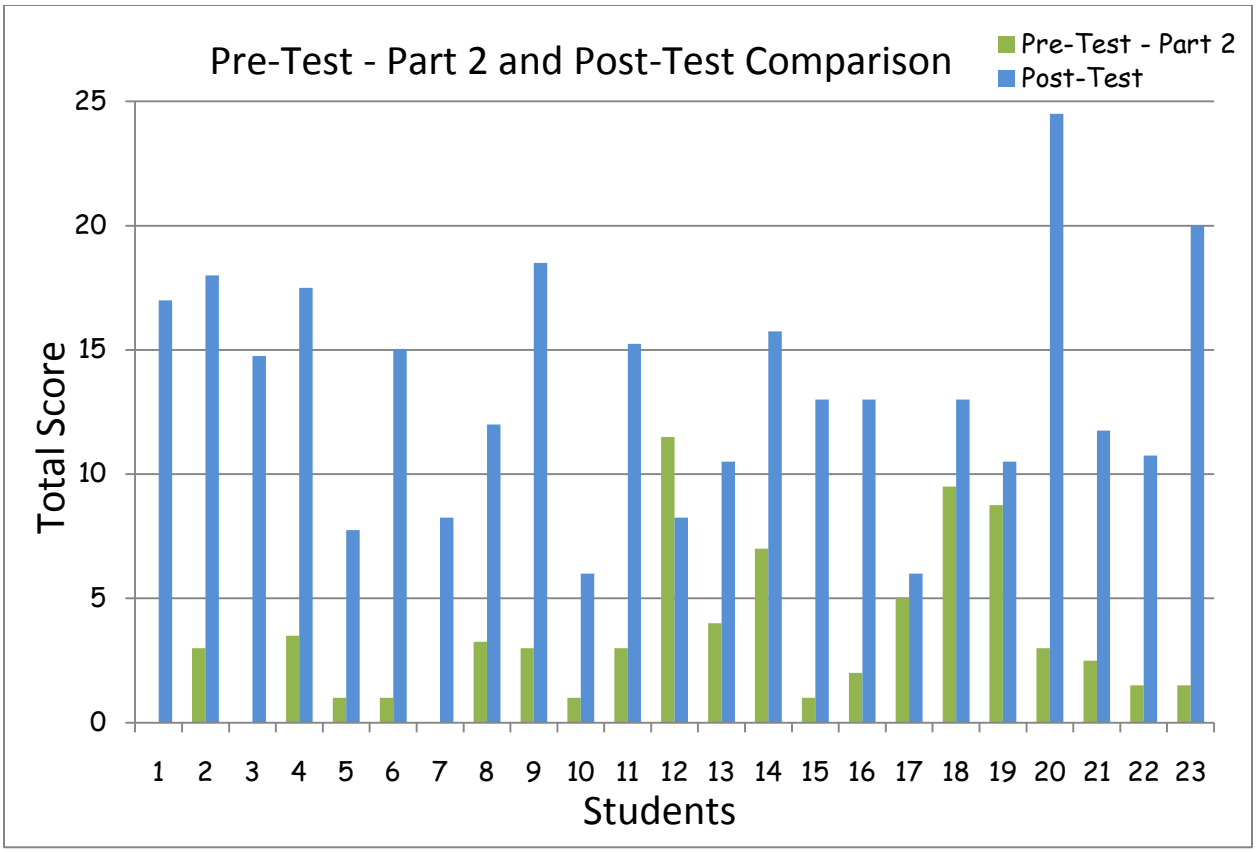


The increase from the pre-test to the mid-test ranged from 3 to 11.25 points. The 11.25 points translates to the highest percentage increase of 59%. The average increase was 6.91 points which calculates to a 36% increase.

4.2 Results from Pre-Test – Part 2 and Post-Test

The data from Pre-Test – Part 2 (Appendix B) and the Post-Test (Appendix D) were compared and analyzed. These two tests examined the students' understanding of inequalities with variables rather than numbers, finding equations to system of inequalities, graphing system of inequalities, and knowing equations in 2 and 3 dimensions. Also included was graphing a line in 2 dimensions and overlaying it with parabolas that initially had variables in the equations which were then reconstructed by the student who decided what number to use as the variable to allow the line and parabolas to intersect. These concepts were learned via activities 4 – 6. The Post Test is identical to the Pre-Test – Part 2. Pre-Test – Part 2 data is colored medium blue and Post-Test data is colored medium green in the graphs.

Pre-Test – Part 2 and the post-test were out of 32 points. On the pre-test, three students had the lowest score of a zero. The highest score was 11.5. The mean was 3.3 and the standard deviation was 3.13. On the post-test, two students had the lowest score of a 6; the highest score was 24.5. The mean was 13.35 and the standard deviation was 4.67.

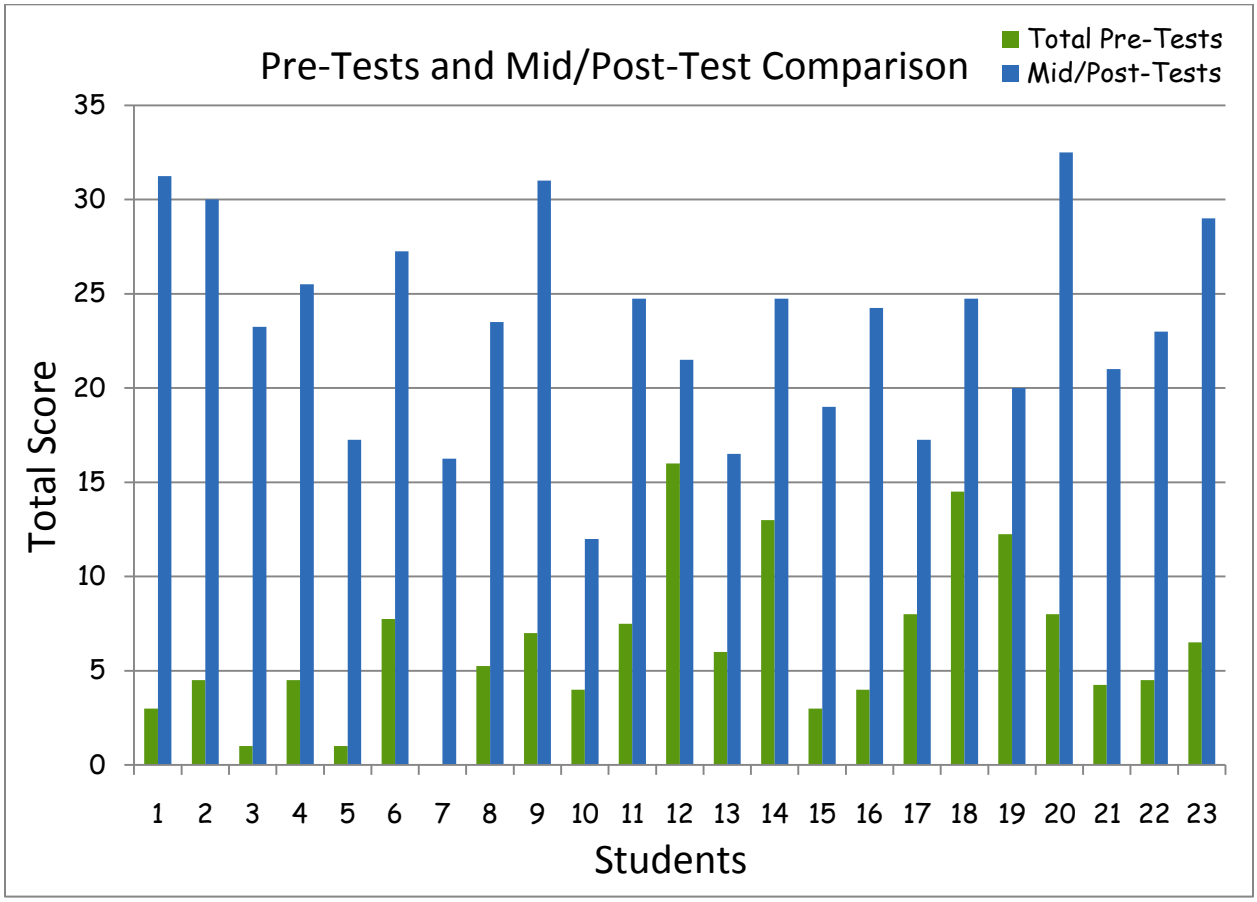


The increase from the pre-test to the post-test ranged from 1 to 21.5 points. Only one student had a decrease in their score by 3.25 points of which there is no obvious explanation. The 21.5 points translates to the highest percentage increase of 67%. The average increase was 10.04 points which calculates to a 31% increase.

4.3 Results from Pre-Tests and Mid/Post-Tests

The data from both pre-tests were combined and the data from the mid-test and the post-test were combined. The results were compared and analyzed. Pre-Tests data is colored dark blue and Mid/Post-Tests data is colored dark green in the graphs.

The combined pre-tests and mid/post-tests were out of 51 points. One student had the lowest pre-tests score of a zero. The highest score was 16. The mean was 6.33 and the standard deviation was 4.24. One student had the lowest score of a 12; the highest score was 32.5. The mean was 23.28 and the standard deviation was 5.42.



The increase from the pre-tests to the mid/post-tests ranged from 5.5 to 28.25 points. The 28.25 points translates to the highest percentage increase of 55%. The average increase was 16.96 points which calculates to a 33% increase.

4.4 Results Grouped by Subject Matter

The data was grouped by subject matter to analyze the data from a different perspective. The subjects of interest comprise of discrete sets, continuous sets, inequalities, 1 dimension, 2 dimensions, 3 dimensions, 1, 2, & 3 dimensions (combined), graphing, and equations. The data for analysis was taken from the pre-tests, mid-test, and post-test that corresponded to the appropriate subjects.

The following table explains how the questions were grouped together by subject. Mid-Test and Post-Test questions were matched with Pre-Test – Part 1 and Pre-Test – Part 2 for comparison. For specific questions, look at the appendices.

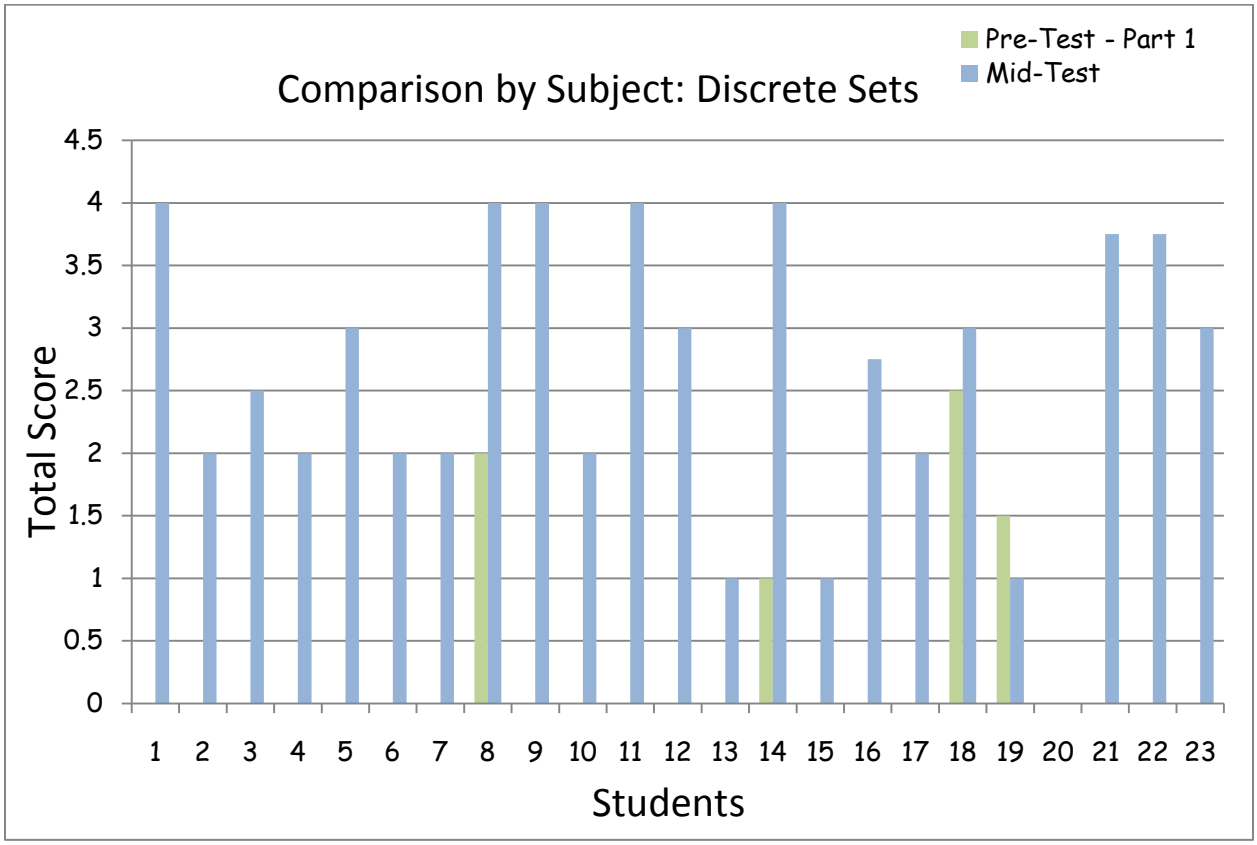
	Question Numbers			
Subject	**Pre-Test 1**	**Pre-Test 2**	**Mid-Test**	**Post-Test**
Discrete Sets	4, 5		4, 5	
Continuous Sets	1, 2, 3		1, 2, 3	
Inequalities	1, 2, 9	1, 2, 3, 4, 5	1, 2, 8	1, 2, 3, 4, 5
1 Dimension	1, 2, 6	1, 2	1, 2, 6	1, 2
2 Dimensions	7, 9	3, 4, 5, 6, 8, 9	7, 8	3, 4, 5, 6, 8, 9
3 Dimensions		7		7
Graphing	1, 2, 9	5, 8, 9	1, 2, 8	5, 8, 9
Equations	6, 7	3, 4, 6, 7, 9	6, 7	3, 4, 6, 7, 9

4.4.1 Discrete Sets

Data was taken from the first pre-test and the mid-test to total 4 points each.

Note that on the pre-test, only four students had a score higher than zero, hence there are no green bars on the chart below, with 2.5 being the highest score. The mean was 0.3 and the standard deviation was 0.72. On the mid-test, only one

student had a score of zero; five students had the highest score of 4. The mean was 2.6 and the standard deviation was 1.16.

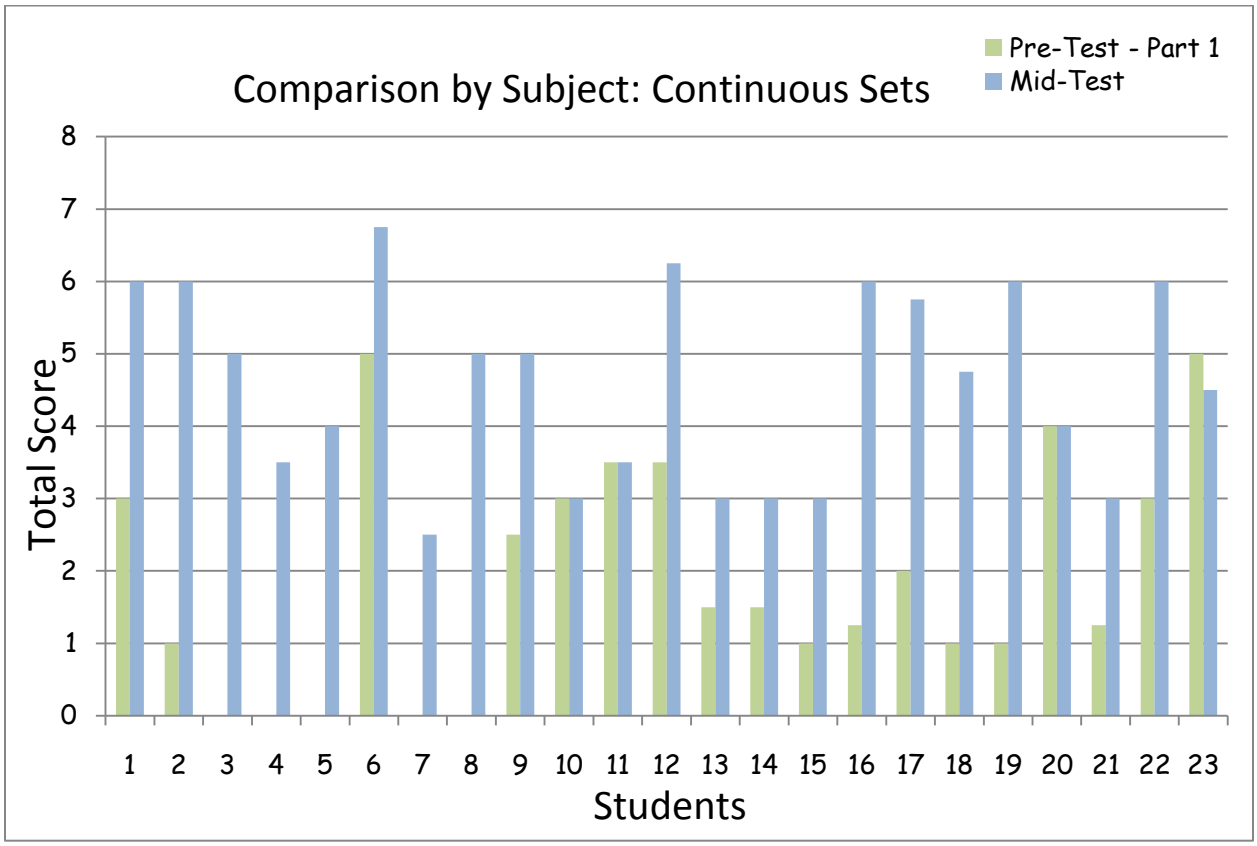


The increase from the pre-test to the mid-test went from 0 to 4 points. The 4 points translates to the highest percentage increase of 100%. The average increase was 2.29 points which calculates to a 57% increase. Only one student's score decreased from 1.5 to 1. There is no known reason for this decrease.

4.4.2 Continuous Sets in the Real Spaces

Data was taken from the first pre-test and the mid-test which were out of 10 each.

Note that on the pre-test, five students (3, 4, 5, 7, 8) had a score of zero while the other scores ranged from 1 to 5. The mean was 1.91 and the standard deviation was 1.58. On the mid-test, the lowest score was 2.5 and the highest score was 6.75. The mean was 4.59 and the standard deviation was 1.34.



The increase from the pre-test to the mid-test went from 0 to 5 points. The 5 points translates to the highest percentage increase of 50%. The average increase

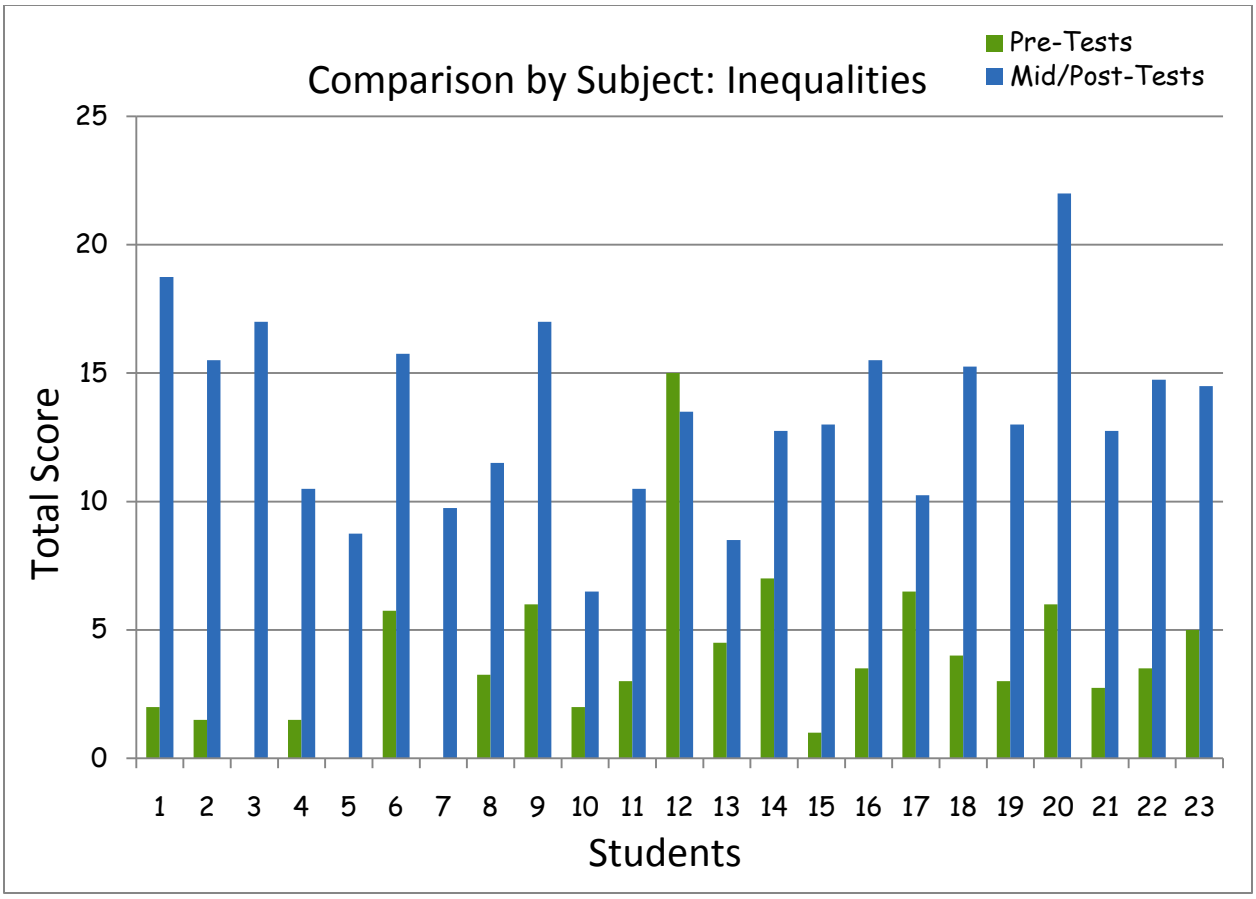
was 2.67 points which corresponds to a 27% increase. Only one student's score decreased from 5 to 4.5. There is no explanation for this negative change.

4.4.3 Inequalities

Data was taken from both pre-tests and from both the mid-test and post-test each totaling 32.5 points.

Note that three students (3, 5, 7) had the lowest pre-test score of a zero. The highest score was 15. The mean was 3.77 and the standard deviation was 3.23. For the mid/post-tests, one student had the lowest score of a 6.5; the highest score was 22. The mean was 13.36 and the standard deviation was 3.61.

The increase range from the pre-tests to the mid/post-tests was from 3.75 to 17 points. The 17 points translates to the highest percentage increase of 52%. The average increase was 9.59 points which calculates to a 29% increase. Only one student showed a 5% decrease in their score going from 15 to 13.5 of which there is no obvious explanation.

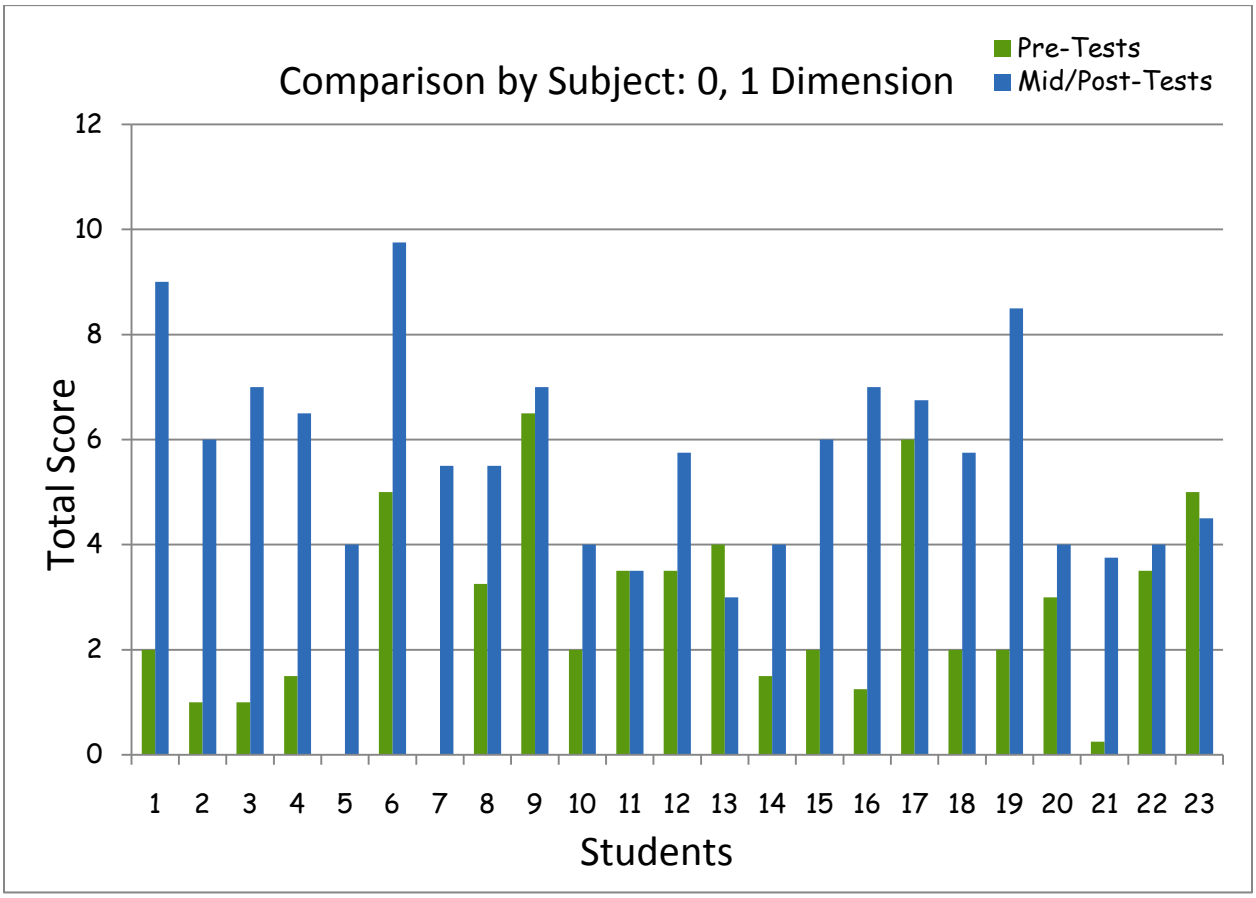


4.4.4 0, 1 Dimension

Data was taken from both pre-tests and the mid-test and post-test to total 14.5 points each.

Note that two students (5, 7) had the lowest pre-test score of a zero. The highest score was 6.5. The mean was 2.6 and the standard deviation was 1.83. For the mid/post-tests, one student had the lowest score of a 3; the highest score was 9.75. The mean was 5.68 and the standard deviation was 1.83.

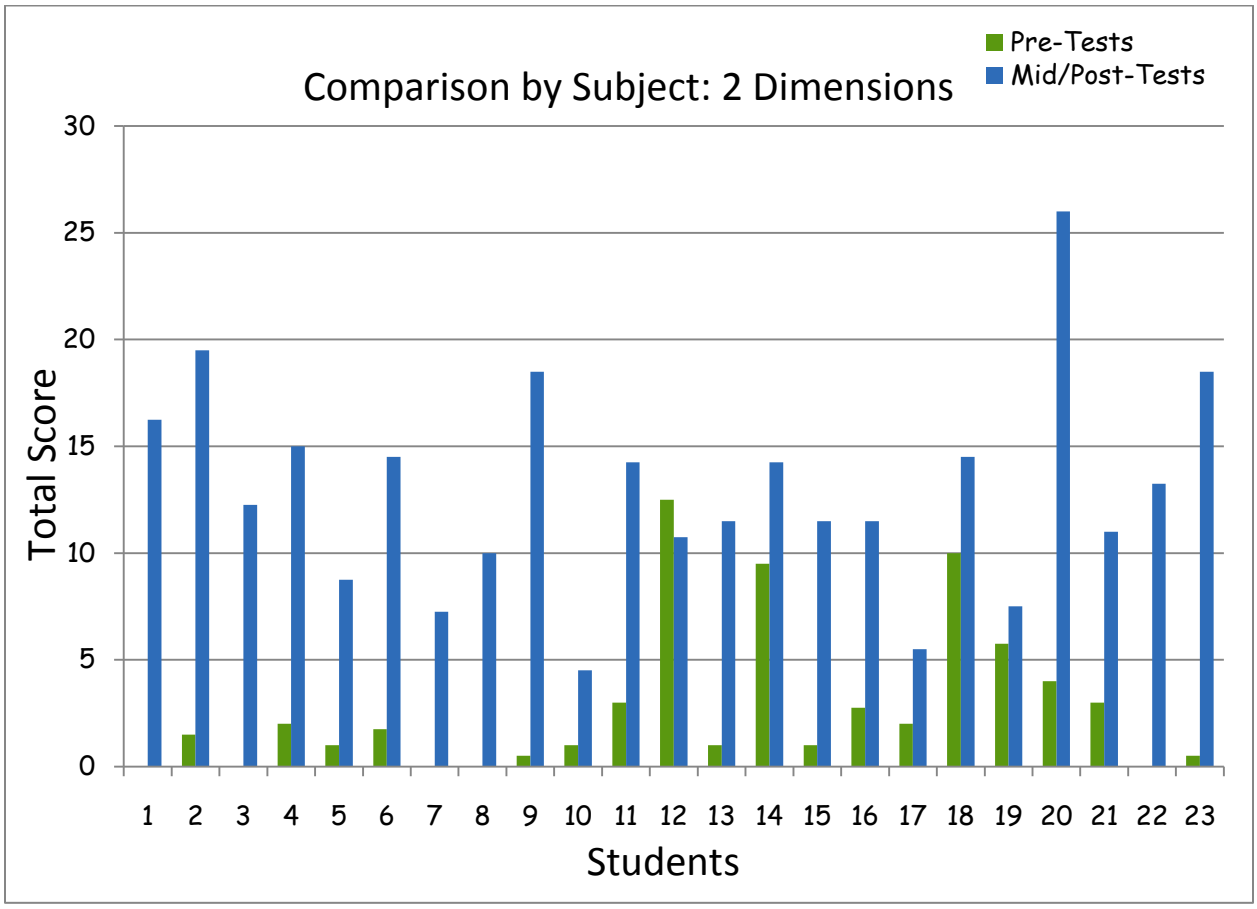
The increase range from the pre-tests to the mid/post-tests was from 0 to 7 points. The 7 points translates to the highest percentage increase of 48%. The average increase was 3.09 points which calculates to a 21% increase. One student showed a 3% decrease in their score going from 5 to 4.5 and another had a 7% decrease from 4 to 3. The students with the 7% decrease was very ill a few weeks out of the semester which could have contributed to the lower score. There is no explanation as to why the other student's score decreased by 3%.



4.4.5 2 Dimensions

Data was taken from both pre-tests and the mid-test and post-test to total 28.5 points each.

Note that five students (1, 3, 7, 8, 22) had the lowest pre-test score of a zero. The highest score was 12.5. The mean was 2.73 and the standard deviation was 3.49. For the mid/post-tests, one student had the lowest score of a 4.5; the highest score was 26. The mean was 12.89 and the standard deviation was 4.9.



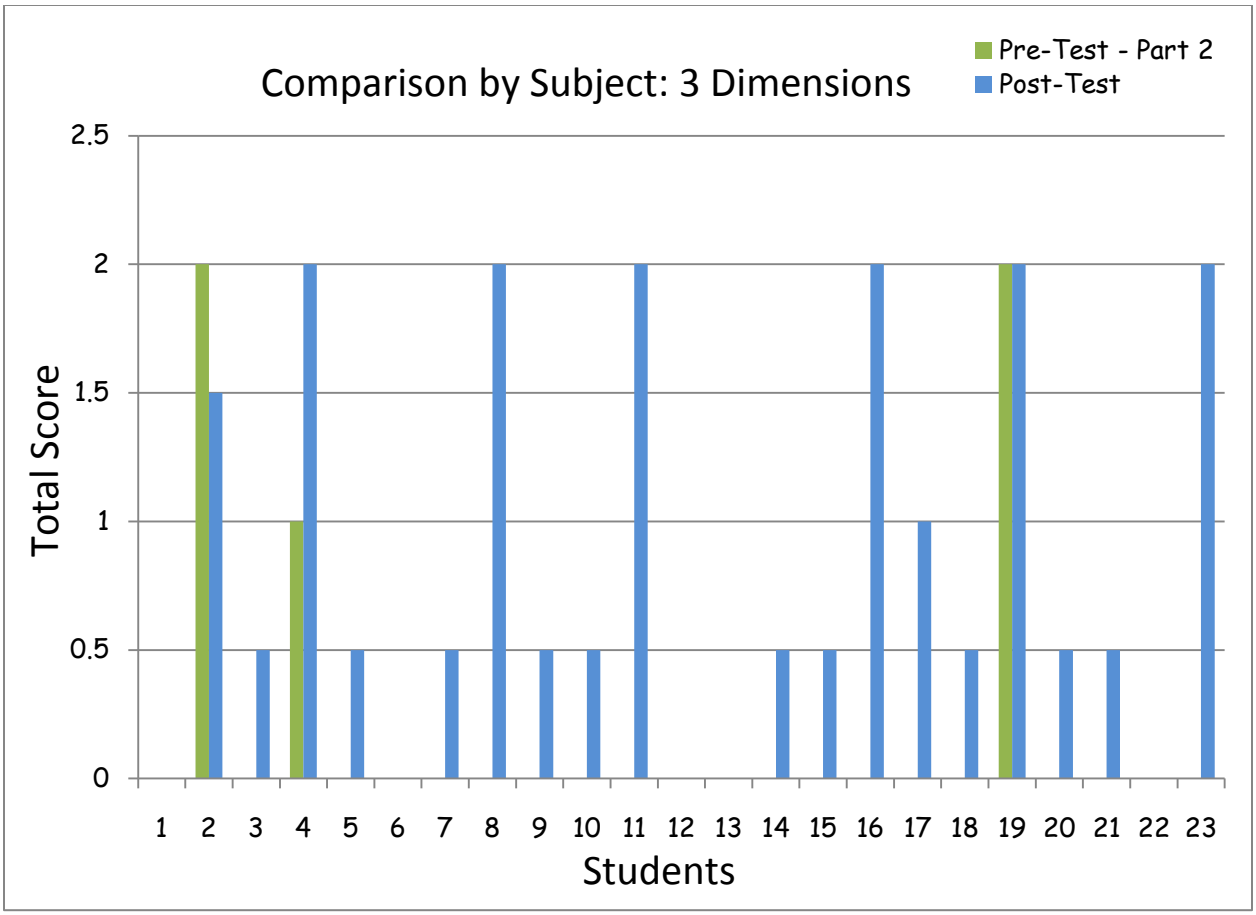
The increase range from the pre-tests to the mid/post-tests was from 1.75 to 22 points. The 22 points translates to the highest percentage increase of 77%. The average increase was 10.16 points which calculates to a 36% increase. Only one student showed a 6% decrease in their score going from 12.5 to 10.75. It is not known as to why this student's score decreased.

4.4.6 3 Dimensions

Data was taken from the second pre-test and the post-test to total 2 points for each test.

On the pre-test, all students scored a zero except for one student who scored a 1 and two students scored a 2. The mean was 0.22 and the standard deviation was 0.6. On the post-test, the lowest score was 0 and the highest score was 2. The mean was 0.85 and the standard deviation was 0.78.

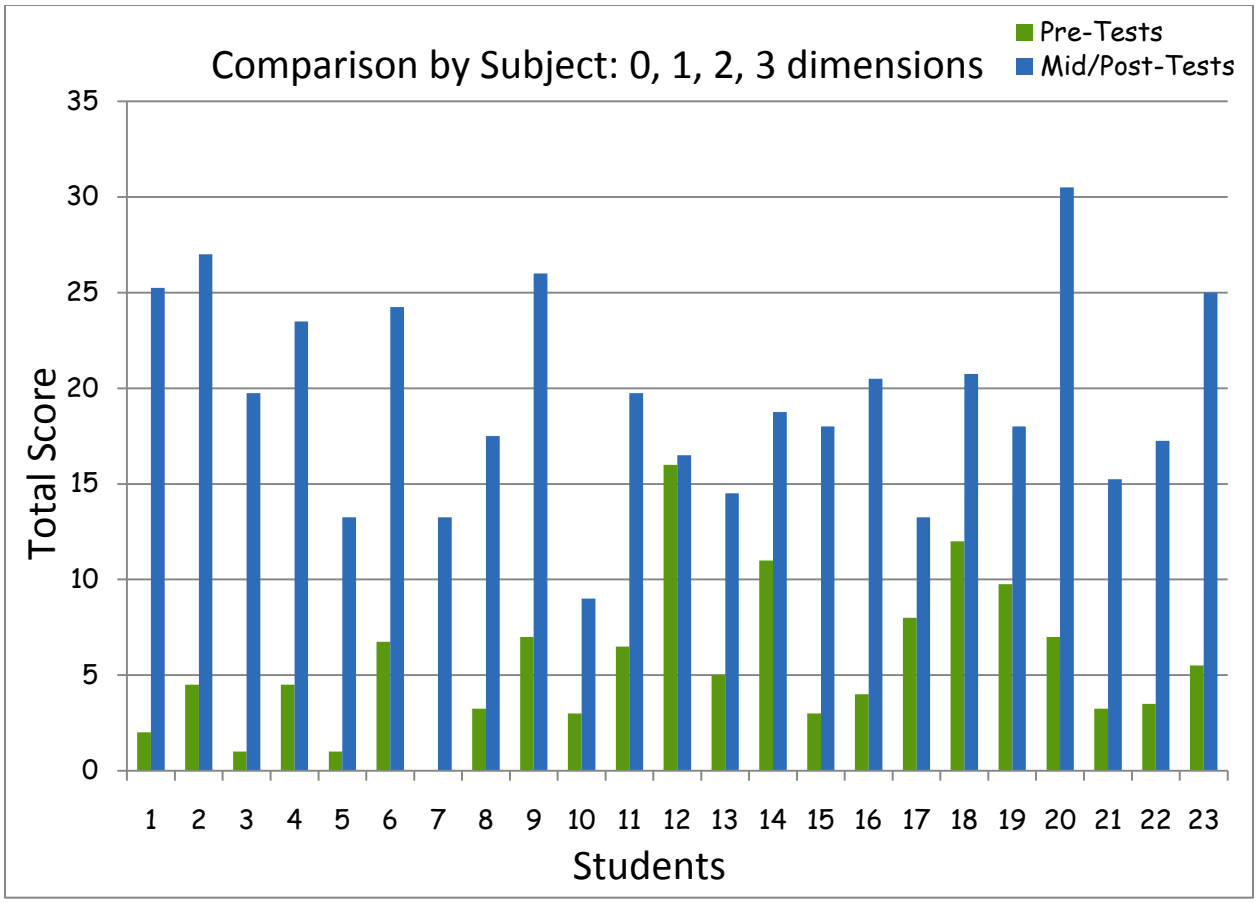
The increase from the pre-test to the post-test went from 0 to 2 points. The 2 points translates to the highest percentage increase of 100%. The average increase was 0.63 points which corresponds to a 32% increase. Only one student's score decreased by 25% from 2 to 1.5. There is no known reason why this student's score decreased.



4.4.7 0, 1, 2, and 3 Dimensions

Data from 0, 1, 2, and 3 dimensions were combined to analyze. The pre-test and the mid/post-tests totaled 45 points each.

Note on the pre-test, only one student (7) scored a zero while the highest score was 16 points. The mean was 5.54 and the standard deviation was 3.86. On the mid/post-tests, the lowest score was 9 and the highest score was 30.5. The mean was 19.42 and the standard deviation was 5.30.

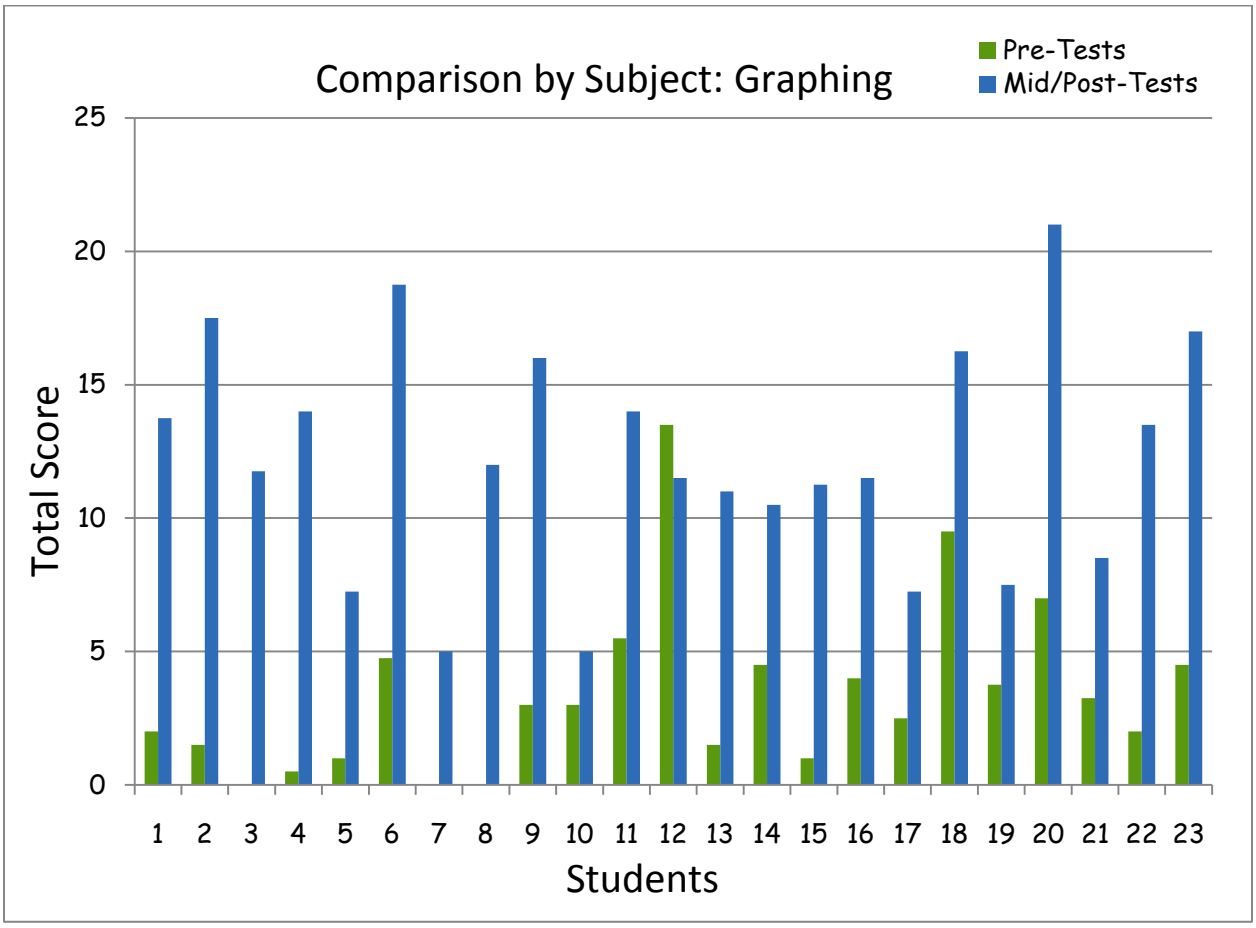


The increase from the pre-tests to the mid/post-tests went from 0.5 to 23.5 points. The 23.5 points translate to the highest percentage increase of 52%. The average increase was 13.88 points which corresponds to a 31% increase.

4.4.8 Graphing

Data was taken from both pre-tests and the mid/post tests to total 28.5 points each.

Note that three students (3, 7, 8) had the lowest pre-test score of a zero. The highest score was 13.5. The mean was 3.4 and the standard deviation was 3.22. For the mid/post-tests, two students had the lowest score of a 5; the highest score was 21. The mean was 12.25 and the standard deviation was 4.31.



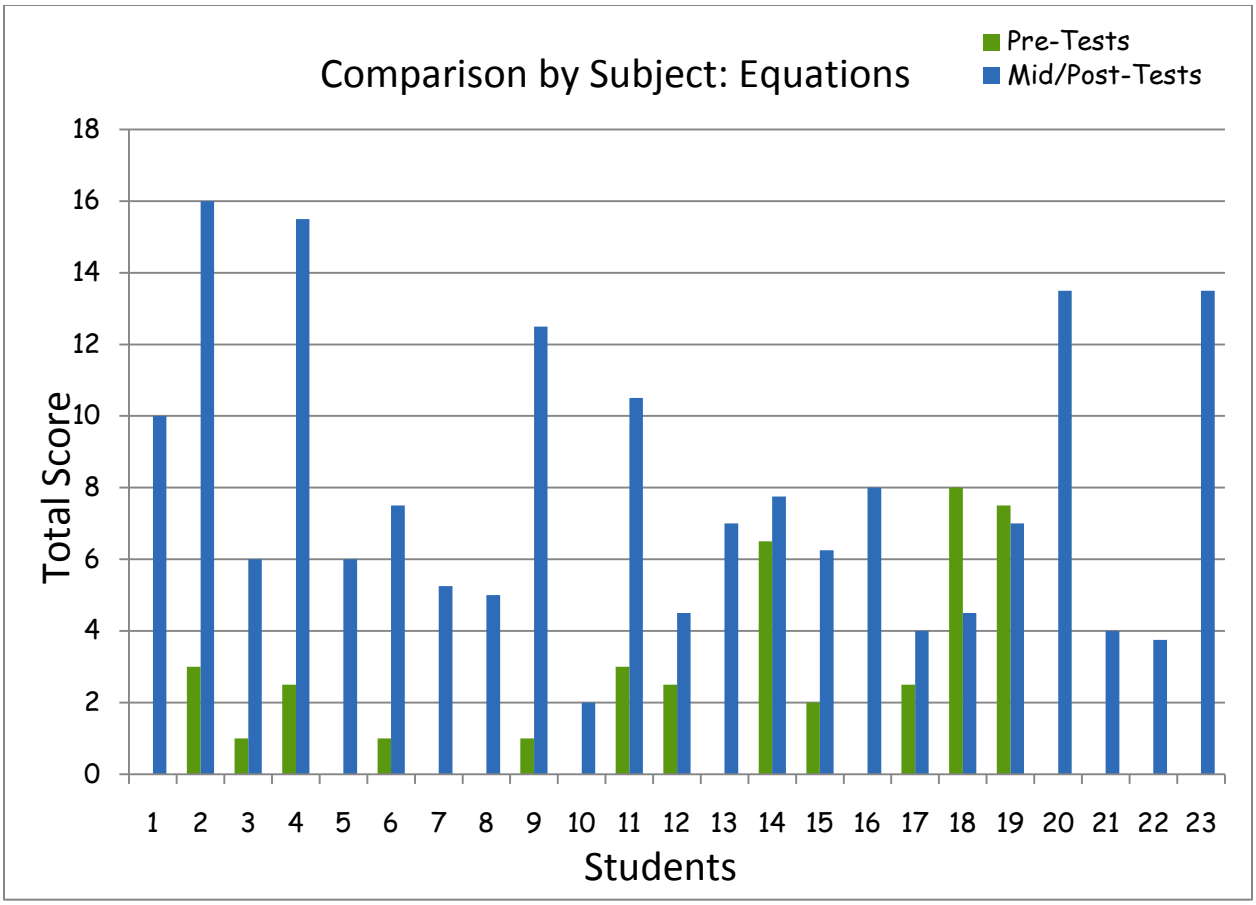
The increase range from the pre-tests to the mid/post-tests was from 2 to 16 points. The 16 points translates to the highest percentage increase of 56%. The average increase was 8.85 points which calculates to a 31% increase. Only one student showed a 7% decrease in their score going from 13.5 to 11.5 of which there is no known explanation.

4.4.9 Equations

Data was taken from both pre-tests and the mid-test and post-test to total 19.5 points each.

About half of the students had the lowest pre-test score of a zero (hence the green bars are missing above). The highest score was 8. The mean was 1.76 and the standard deviation was 2.48. For the mid/post-tests, one student had the lowest score of a 2; the highest score was 16. The mean was 7.83 and the standard deviation was 3.99.

The increase range from the pre-tests to the mid/post-tests was from 1.25 to 13.5 points. The 13.5 points translates to the highest percentage increase of 69%. The average increase was 6.07 points which calculates to a 31% increase. Two students showed a decrease in their scores: a 3% decrease from 7.5 to 7 and an 18% decrease from 8 to 4.5. There is no known reason why the scores of these two students decreased.


Comparison by Subject: Equations
Pre-Tests
Mid/Post-Tests
Total Score
0
2
4
6
8
10
12
14
16
18
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Students

4.5 Statistical Analysis

The following tables show the results from 4.1 – 4.4.

						Point Increase from pre-test to mid/post-test			Number of Students whose Score…			Average Increase	
By Test	Total Points	Low	High	Mean	Standard Deviation	Low	High	Max Percent Increase	De-creased	In-creased	Stayed the Same	Points	%
Pre-Test - Part 1	19	0	6.75	3.02	1.84								
Mid-Test	19	6	14.25	9.93	2.36	3	11.25	59%	0	23	0	6.91	36%
Pre-Test - Part 2	32	0	11.5	3.30	3.13								
Post-Test	32	6	24.5	13.35	4.67	-3.25	21.50	67%	1	22	0	10.04	31%
Both Pre-Tests	51	0	16	6.33	4.24								
Mid/Post -Tests	51	12	32.5	23.28	5.42	5.50	28.25	55%	0	23	0	16.96	33%

						Point Increase from pre-test to mid/post-test			Number of Students whose Score…			Average Increase	
By Subject (pre and mid/post)	Total Points	Low	High	Mean	Standard Deviation	Low	High	Max Percent Increase	De-creased	In-creased	Stayed the Same	Points	%
Discrete Sets	4	0	2.5	0.30	0.72								
	4	0	4	2.60	1.16	-0.50	4.00	100%	1	21	1	2.29	57%
Continuous Sets	10	0	5	1.91	1.58								
	10	2.5	6.75	4.59	1.34	-0.50	5.00	50%	1	19	3	2.67	27%
Inequalities	32.5	0	15	3.77	3.23								
	32.5	6.5	22	13.36	3.61	-1.50	17.00	52%	1	22	0	9.59	29%
0, 1 Dimensions	14.5	0	6.5	2.60	1.83								
	14.5	3	9.75	5.68	1.83	-1.00	7.00	48%	2	20	1	3.09	21%
2 Dimensions	28.5	0	12.5	2.73	3.49								
	28.5	4.5	26	12.89	4.90	-1.75	22.00	77%	1	22	0	10.16	36%
3 Dimensions	2	0	2	0.22	0.60								
	2	0	2	0.85	0.78	-0.50	2.00	100%	1	16	6	0.63	32%
0, 1, 2, 3 Dimensions	45	0	16	5.54	3.86								
	45	9	30.5	19.42	5.30	0.50	23.50	52%	0	23	0	13.88	31%
Graphing	28.5	0	13.5	3.40	3.22								
	28.5	5	21	12.25	4.31	-2.00	16.00	56%	1	22	0	8.85	31%
Equations	19.5	0	8	1.76	2.48								
	19.5	2	16	7.83	3.99	-3.50	13.50	69%	2	21	0	6.07	31%

As mentioned at the beginning of this chapter (pg 31), a t-test was performed on the data using a right-sided, one-tailed α-value of 5% or 0.05. The results compared the calculated p-value against 0.05 with the degree of freedom of 22. If the p-value is less than α, then we have strong evidence to reject the null hypothesis and accept the alternative hypothesis. The null hypothesis equals the mean of post-test/pre-test differences is less than or equal to 0 and the alternative hypothesis states that the mean of post-test/pre-test differences is greater than 0.

Data Analysis by Tests	t - Statistic	p - Value
Pre-Test - Part 1 & Mid-Test	12.98	< 0.0001
Pre-Test - Part 2 & Post-Test	8.01	< 0.0001
Total Pre-Tests & Mid/Post-Tests	12.89	< 0.0001

As we can see from the above chart, we should reject each null hypothesis in favor of its alternative hypothesis. Therefore, when the data is grouped and compared by tests, we can say with 95% confidence that the increase in the final scores were significant enough to conclude that the six activities helped the students learn considerably.

Now look at the data grouped by subject. The general conclusion is the same but let's look at one example to illustrate what is happening with all scenarios.

Data Analysis by Subject	t - Statistic	p - Value
Discrete Sets	8.66	< 0.0001
Continuous Sets	7.32	< 0.0001
Inequalities	10.49	< 0.0001
0, 1 Dimension	6.19	< 0.0001
2 Dimensions	8.21	< 0.0001
3 Dimensions	4.16	0.0002
1, 2, & 3 Dimensions	10.90	< 0.0001
Graphing	9.39	< 0.0001
Equations	6.19	< 0.0001

Here our calculated t-statistic is equal to 1.717. For the grouping of continuous sets, we see that the t-statistic is 7.32 > 1.717 but more importantly, the p-value is less than 0.0001 < 0.05. Therefore, this constitutes strong evidence that the null hypothesis is false and we can be 95% confident that the alternative hypothesis is likely to be true hence the activities improved student performance.

4.6 Summary of the Data

We can confirm through the statistical analysis, including the p-values and t-statistics, that there is significant evidence that students' performance on certain Intermediate Algebra concepts increases through the use of our activities with manipulatives.

4.7 Bias

There is a possibility of a bias that could have affected the outcome of this study. The pretests were administered on the first and second day of class. Since this class was in the fall, the likelihood of the students studying any kind of math in the summer was slim. That may explain why the pretests were so low.

Since we did not have a control group for this study the design of this experiment could bring some bias since there was no student group to compare our students to. Some of the other factors that could have affected the results may include the following student characteristics reported on the survey:

- Morning person: 26%

  Those who were morning persons could have been more willing and able to concentrate on the activities.

- Had a tutor: 22%

  Since the activities were scheduled for Fridays except for one Monday, the only test that could have been affected is the mid-test and the post-test.

- Taken this class before: 23%

  Being familiar with most of the material taught via the activities could have led to a bias in the results.

- Doesn't hate math: 39%

  The students who don't hate math may have been more willing and open to learning more difficult concepts such as 3 dimensions.

- Family members attended college: 52%

The family members who attended college may have helped tutored the students.

- Took a prior math class at the Algebra level or higher: 48%
  The prior knowledge that these particular students had may have been refreshed during the activities.

- Worked 0 – 5 hours a week: 44%
  The students who had more time on their hands may have been able to focus more on the activities than those who had more responsibilities to think about.

- Stressed level very low or low: 9%
  Very few students had a low stress level. Even so, this 9% could have biased the results.

- High school GPA 3.5 or above: 35%
  A third of the students are considered quite smart and/or good at test taking. This could have skewed the data analysis toward positive results.

- Grade in prior math class was A or B: 35%
  Since a third of the students did well in their prior math class, they may be good at math. If so, their abilities rather than the activities could have increased their test scores.

- Students' majors that require math: 26%
  A quarter of the students need math for their major so there is a good chance that they look at math as something they are willing to delve into and explore.

In addition, even though the students in this study were chosen randomly by the university scheduling system, there were students who needed further help to understand the Algebra concepts required for entering the regular university program. Also, the fact that the same instructor taught all activities as well as the regular class material may cause some bias, as personality of the instructor could have played a role. Teaching style, including possible one-on-one help, and additional learning and assistance from the required math computer lab could have altered the results.

4.8 Student Feedback

The students communicated their thoughts on what their confidence level was for each question on each pre-test, the mid-test, and the post-test. The scale below was used for each problem.

Confidence Level	
1	Very Confident
2	Confident
3	Slightly Confident
4	Neutral
5	Slightly Unsure
6	Unsure
7	Very Unsure
8	I felt totally lost
9	Other

The students filled out the same type of questionnaire for each activity. This questionnaire focused on how the students felt about the related subject matter in that activity.

This data from the questionnaires were compared to find out if the students' confidence increased or decreased and by how much.

The chart below shows that the confidence level increased (in absolute value) for all questions (all went up). For Pre-Test – Part 1 and Mid-Test, the students had an average confidence increase of 4 confidence levels for question 4. For this particular problem, that meant that for one student, the confidence level went from 8 ("I felt totally lost.") to 3 (Slightly Confident") jumping five confidence levels. Another student for the same problem went from 7 ("Very Unsure") to 4 (Neutral) jumping three confidence levels. And yet went from 9 ("Other") to 2 (Confident") jumping seven confidence levels. Overall, this average increase could mean the students were *unsure* on how to work out this problem on the pre-test but when they saw that question again on the mid-test, they were *confident* on how to work the problem through.

Confident Level from Pre-Tests 1 to Mid/Post-Tests						
	Pre-Test - Part 1 to Mid-Test			Pre-Test - Part 2 to Post-Test		
	Average	Confidence Range		Average	Confidence Range	
Questions	Increase	Pre-Test	Mid-Test	Increase	Pre-Test	Post-Test
Q1	1.22	2 to 9	1 to 8	2.09	1 to 9	1 to 8
Q2	1.65	1 to 9	1 to 4	2.00	1 to 9	1 to 9
Q3	2.43	2 to 9	1 to 5	2.85	3 to 9	1 to 8
Q4	4.26	3 to 9	1 to 8	1.87	4 to 9	1 to 9
Q5	3.48	2 to 9	2 to 8	2.04	3 to 9	1 to 9
Q6	3.04	1 to 9	1 to 9	4.09	1 to 9	1 to 9
Q7	2.87	1 to 9	1 to 9	3.48	1 to 9	1 to 9
Q8	1.78	1 to 9	1 to 9	2.30	1 to 9	1 to 9
Q9a	-	-	-	1.87	2 to 9	1 to 9
Q9b	-	-	-	2.13	2 to 9	1 to 9
Q9c	-	-	-	2.35	4 to 9	1 to 9

Remember that for the range, the lower the number the more confident the student was (see Confidence Level chart on page 53).

Now let's look at the confidence level increase for each activity.

Average Increase in Confident Level by Activity									
	Activity 1			Activity 2			Activity 3		
	Average	Confidence Range		Average	Confidence Range		Average	Confidence Range	
Question	Increase	Before	After	Increase	Before	After	Increase	Before	After
Q1	1.39	1 to 6	1 to 5	1.30	1 to 7	1 to 4	1.74	2 to 7	1 to 6
Q2	2.70	2 to 7	1 to 3	1.35	1 to 7	1 to 5	2.13	2 to 7	1 to 6
Q3	2.83	2 to 7	1 to 3	1.61	1 to 7	1 to 6	1.91	2 to 8	1 to 7
Q4	0.96	1 to 6	1 to 4	2.26	2 to 7	1 to 4	1.61	2 to 8	2 to 8
Q5	0.91	2 to 7	1 to 7	2.52	2 to 7	1 to 4	2.26	2 to 8	1 to 6

Average Increase in Confident Level by Activity									
	Activity 4			Activity 5			Activity 6		
Question	Average Increase	Confidence Range Before	Confidence Range After	Average Increase	Confidence Range Before	Confidence Range After	Average Increase	Confidence Range Before	Confidence Range After
Q1	1.39	2 to 7	1 to 4	0.83	2 to 6	1 to 6	0.65	2 to 6	1 to 5
Q2	1.48	2 to 7	1 to 5	0.65	2 to 7	1 to 6	0.91	2 to 8	1 to 5
Q3	1.52	1 to 7	1 to 4	0.96	2 to 7	2 to 8	1.39	2 to 8	1 to 5
Q4	1.74	2 to 7	1 to 4	0.96	2 to 6	1 to 6	1.43	2 to 8	1 to 5
Q5	1.78	2 to 7	1 to 4	1.13	2 to 7	2 to 7	1.63	2.5 to 8	2 to 5

Again, we see that average confidence level increased for all questions. The activity that had the most impact on confidence level was activity 3 (Playdough - Graphing) and the activity that had the least impact on confidence level was activity 5 (Graph Blast Bingo - Equations).

Activity 5 focused on figuring out the equation to a graph. This can be a challenge to most people, requiring extra thinking. That could explain why the confident level did not increase like the others.

After each activity, students made comments in questionnaires about what they learned and how the activities helped or did not help in the learning process. Some of the comments students relayed about the activities are as follows:

<u>Activity 1: Venn Diagrams</u>

- "The most helpful thing was with the different colors and shapes and putting them together. Least helpful, nothing, everything was good."

- "The best thing was interacting and using the objects because it made it more clear."
- "The visuals used in showing what a union and intersection is really helped me to memorize how to do the problem."
- "The colors of the objects and the Venn diagram is very helpful."
- "Using my own info was helpful in that I couldn't just mindlessly write down. The colors kind of threw me off."
- "What was most helpful was the definitions of intersections and unions. This activity was really confusing at first."

<u>Activity 2: Bamboo Skewers and Money</u>

- "The colors helped me visualize."
- "The overlap of the two colors really helps to understand the intersection and union. No negative comments"
- "It is all pretty confusing to me but my memory will be refreshed as time goes on. The colors overlapping on the skewers was a little bit helpful."
- "The most helpful was when we transferred our inequalities to paper."
- "It was all very helpful"

- "I already knew how to do this, but the activity was really fun. This would be a helpful activity when first introducing overlapping inequalities."
- "The most helpful was the coloring on the skewer and least was nothing really. I understood well."

Activity 3: Playdough

- "It was distracting for me to use the playdough."
- "The playdough helps understanding 3-D graphing."
- "The 3-D view of the sphere was helpful."
- "The most helpful was actually physically working with the playdough and skewers."
- "This felt confusing, I would much rather we had written this out on the board without playdough, like how we'd be doing homework with no sticks or playdough."
- "Most helpful: 3-dimensionals. Least helpful: the graphs."
- "It was helpful being able to visualize the graphs."
- "What was most helpful for me was seeing your examples on the board and applying it to my data."

Activity 4: Transparencies

- "Shading was easy with colors. Plotting correct points was harder."
- "The hardest thing about inequalities is solving and graphing correctly but I even found that easy."
- "The hardest part about inequalities is figuring out whether it's above or below when shaded."
- "The activity was very useful and I understand shading more."
- "The hardest part was solving for x and y. The easiest was figuring out which direction to shade in. The transparencies really helped."
- "Sometimes solving the inequality equation is hard or when there are a lot of lines I get confused."

Activity 5: Graph Blast Bingo

- "Keeping a fast pace helped me focus on identifying the graphs faster."
- "Just seeing examples and trying to figure them out help me to remember them."
- "The activity gave a lot of examples."

- "The graphing bingo was helpful. The least helpful thing was that the answer was not shown after the problem so I don't know if I got the answer right or wrong."
- "It helped going over the answers at the end and explaining them."
- "The colors and color keys were very helpful. I had a hard time determining the right graph though."
- "Visually seeing what the equations looked like on a graph helped. We went really fast so that made it more difficult."

Activity 6: Aquarium

- "The aquarium was very helpful."
- "The most helpful was finding the fish game. The least was making the tank."
- "Using the game helped me better understand the plane."
- "It was helpful to see things in depth rather than just a picture."
- "The most helpful thing for me was putting the graphing paper on the tank."
- "Finding the coordinates for the fish was helpful."
- "Putting the gridline paper up to the tank in order to find the location of the fish was helpful."

- "The 3-D model really helped."
- "The aquarium part was really fun."
- "It helped being able to see and visualize what I was doing instead of imagining it."
- "Finding the fish in the aquarium was the most helpful part."

Some Overall Thoughts

- "And for my overall thoughts of math, I was actually excited to continue in math after your class. And for your activities I thought they helped give a deeper understanding to the material if the original material seemed confusing." This student is a liberal arts major and plans to teach elementary school someday.
- I suggested to seven students that they major in math because they did so well in my class. Of the seven, four had an undeclared major. Two of those four decided to major in math. One of the seven was majoring in communications but decided at the end of the semester to add a minor in math. For the two remaining students, one was a pre-nursing major and the other an art major. The art major planned to study math and computer science along-side the art to someday create video games as a career.

- I had another student change his major from history to computer science. At the time of this writing, he is currently enrolled in Mathematical Thinking.

Chapter 5 - Conclusion and Recommendations

5.1 Validity of Results

This thesis explored the idea that the manipulative teaching technique would help college-age students to understand certain Intermediate Algebra concepts. The statistical analysis and questionnaires support this. Specifically, after implementing the six activities discussed in this paper, there is sufficient evidence of improved performance on sets, inequalities, graphing, and 3 dimensions. Additionally, our surveys show that students' attitude improved toward mathematics and, for some, this increased their desire to take higher level courses.

To also support the validity of these results, the passing rate of the students in this study was compared with the passing rate of the other Intermediate Algebra classes during the same semester. All the Intermediate Algebra classes share a common final. The passing rate for the students who participated in this study had a 91.3% passing rate. The overall passing rate for fall 2010 was 82%. The high passing rate of 91.3% could be a result of the activities although it is not known for sure as the rigorous testing of the other sections was not a part of this study.

The question of the validity of the results is something to consider. Since the author of this thesis was also the instructor, the overseer of the activities, and

the one analyzing the data, it would be recommended to see this experiment repeated with a different instructor.

Another concern is the fact that the students were learning most of these concepts as part of the curriculum in the Intermediate Algebra class. Most subjects, but not all, were taught via the activities first. More precision for future studies would be beneficial. Another idea is to integrate this study with a group of students who has not yet learned or seen these mathematical concepts and will not be learning them in their current enrolled math course.

The last concern is the fact that this experiment was carried out in the fall semester. All the students were freshman so it is highly unlikely that they studied math over the summer. It would be recommended that this study be performed in the spring semester to see if the pretests have higher scores overall for the class. With everything else being the same, the results could be altered.

5.2 Additional Investigation

The positive results of this thesis invite further investigation. Having a control group, a group of students who do not participate in the activities, may give more evidence and insight as to how helpful the activities are in learning. The activities themselves could be adjusted to improve their effectiveness by looking at the student comments and detailed analysis from this experiment.

5.3 Conclusion and Recommendations

The results show that the manipulative technique, using these six activities, is helpful in learning sets, inequalities, graphs, and 3 dimensional thoughts. As a result, these activities are recommended to be used as a supplemental learning tool when teaching these mathematical concepts. They can be taught to students of any age who are learning Intermediate Algebra.

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Appendices

Appendix A – Pre-Test – Part 1

Thesis Pretest

Part 1 – Concrete, 1-2 dimensions

Name _____

Refer to these descriptions when answering the "level of difficulty" problems.

1 = Very Easy 2 = Easy 3 = Average 4 = Challenging but Manageable
5 = Challenging but Barely Manageable 6 = Difficult 7 = Very Difficult
8 = It's so hard I don't know where to begin but I'll try to write something. 9 = I give up!

Show Your Work

1. Graph $3x + 5 \geq x$

(2 points each answer)

Interval Notation: _____ Set-Builder Notation: _____

Circle the level of difficulty of question 1.

1 2 3 4 5 6 7 8 9

2. Graph $-3x \geq 9$

(2 points)

Circle the level of difficulty of question 2.

1 2 3 4 5 6 7 8 9

3. Are the above two graphs discrete or continuous sets? (1 point) _____

How do you know? (1 point) _____

Circle the level of difficulty of question 3.

1 2 3 4 5 6 7 8 9

4. Find the intersection and union. (1 point each answer)

{a, b, c} ∩ {a, x} = _____

{a, b, c} ∪ {a, x} = _____

Circle the level of difficulty of question 4.

1 2 3 4 5 6 7 8 9

5. Are the above two sets discrete or continuous? (1 point) _____

How do you know? (1 point) _____

Circle the level of difficulty of question 5.

1 2 3 4 5 6 7 8 9

6. What is an example of an equation or inequality in 1-dimension? (1 point)

_____

Circle the level of difficulty of question 6.

1 2 3 4 5 6 7 8 9

7. What is an example of an equation or inequality in 2-dimensions? (1 point)

_____

Circle the level of difficulty of question 7.

1 2 3 4 5 6 7 8 9

8. What is a tangent line on y = x^2 ? (5 points) _____

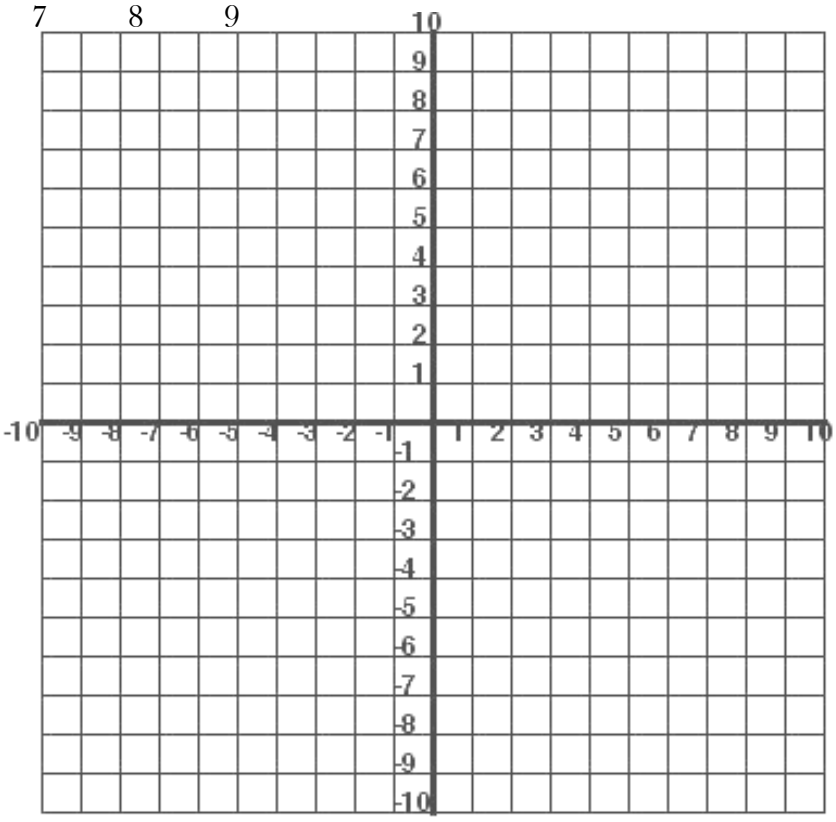
How did you figure out your answer? (2 points)_____

_____

Circle the level of difficulty of question 8.

1 2 3 4 5 6 7 8 9

9. Graph y < 2x – 1 (3 points)



Circle the level of difficulty of question 9.

1 2 3 4 5 6 7 8 9

Appendix B – Pre-Test – Part 2

Thesis Pretest

Part 2 – Abstract, 2-3 dimensions

Name _____

As in part 1, refer to these descriptions when answering the "level of difficulty" problems.

1 = Very Easy 2 = Easy 3 = Average 4 = Challenging but Manageable
5 = Challenging but Barely Manageable 6 = Difficult 7 = Very Difficult
8 = It's so hard I don't know where to begin but I'll try to write something. 9 = I give up!

Show Your Work

1. For all x, $x^2 \geq -a^2$. True False

Why? (2.5 points)

_____

_____

_____

_____

Circle the level of difficulty of question 1.

1 2 3 4 5 6 7 8 9

2. If αx $> a^2$ then x $> \alpha$. Is this always true? Explain your answer. (3 points)

_____

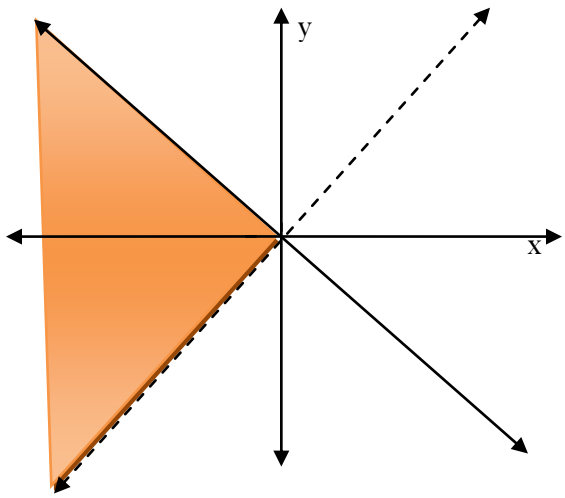
_____

_____

Circle the level of difficulty of question 2.

1 2 3 4 5 6 7 8 9

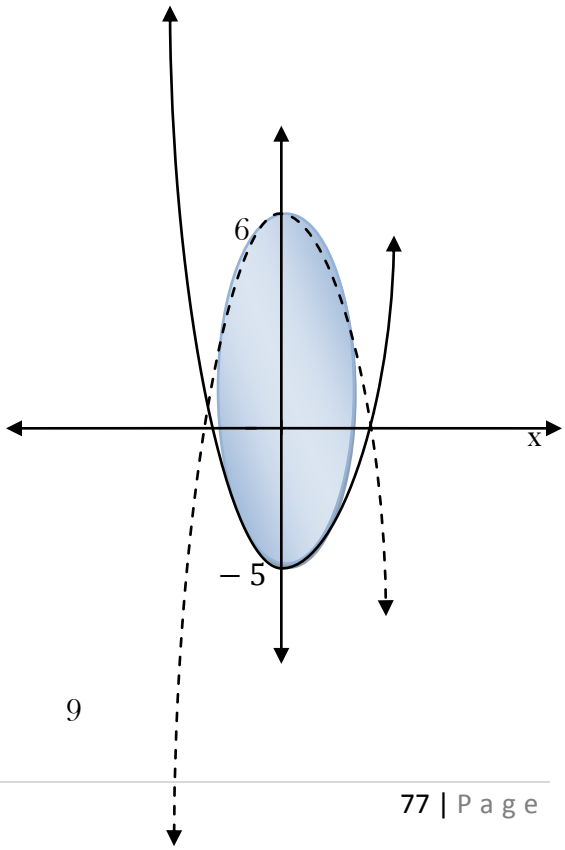
3. What is the system of inequalities that represents this graph? (3 points)



Circle the level of difficulty of question 3.

1 2 3 4 5 6 7 8 9

4. What is the system of inequalities that represents this graph? (3 points)

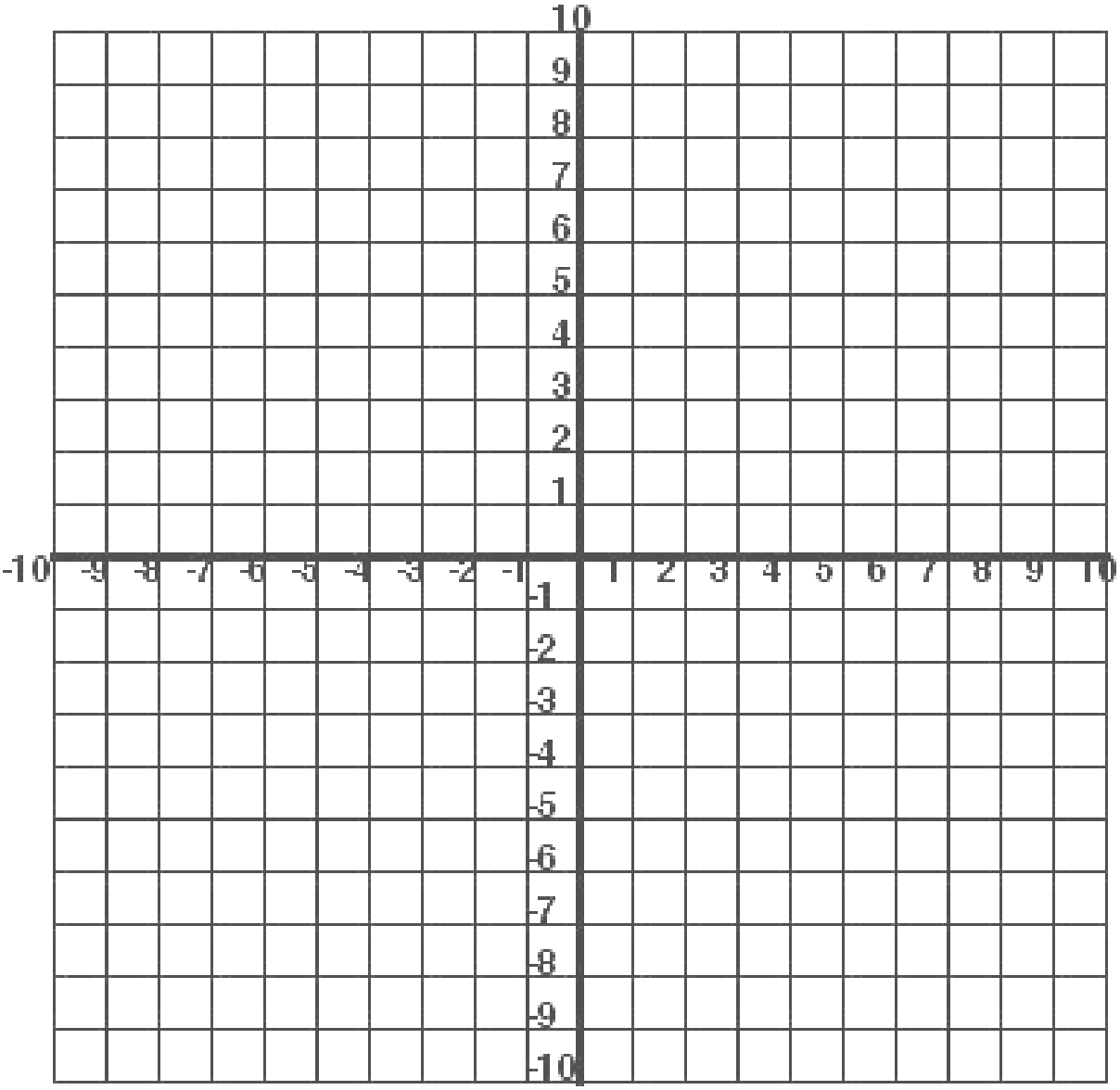


Circle the level of difficulty of question 4.

1 2 3 4 5 6 7 8 9

5. Graph the system of inequalities. Find the coordinates of any vertices (corners) formed. Make sure your graphs are as clear and accurate as possible. (8 points)

$-4x + 2y - 8 \geq 0$ $\quad -y > 2x$ $\quad x > -5$ $\quad y \geq 0$



( ), ( ), ( ), and ( )

Circle the level of difficulty of question 5.

1 2 3 4 5 6 7 8 9

6. What is an example of an equation or inequality in 2-dimensions? (1 point)

_____

Circle the level of difficulty of question 6.

1 2 3 4 5 6 7 8 9

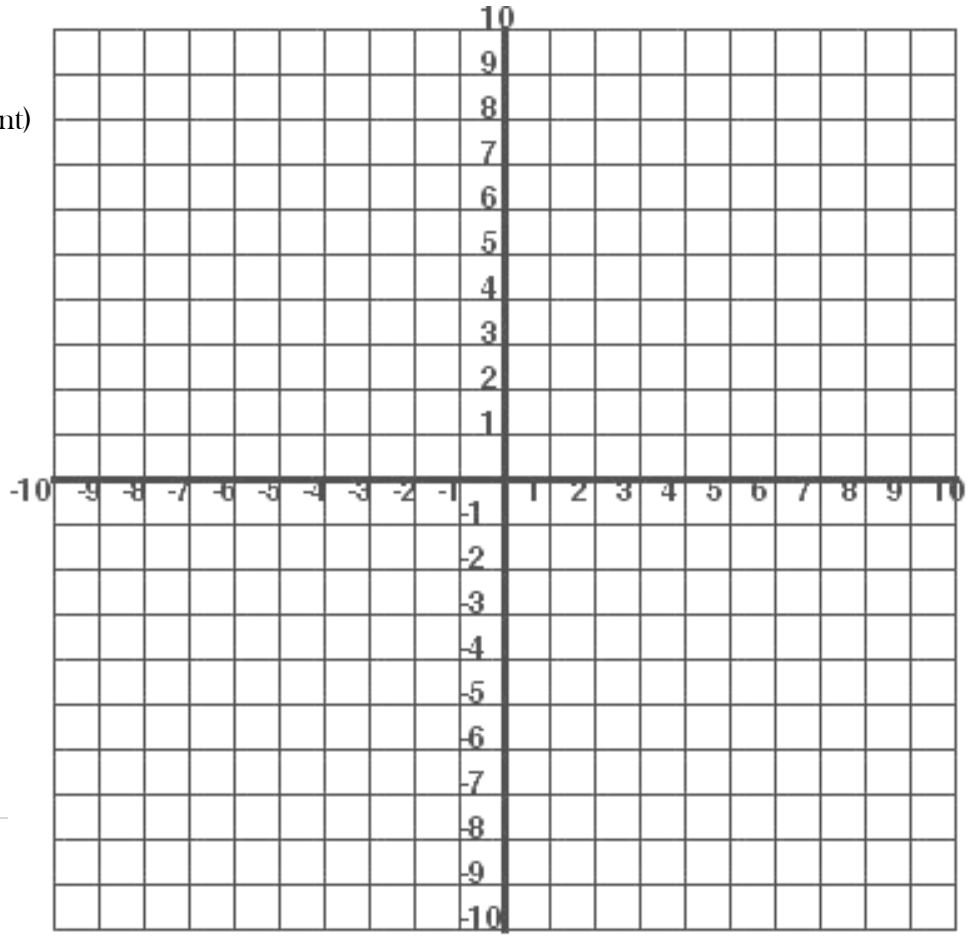
7. What is an example of an equation or inequality in 3-dimensions? (2 points)

_____

Circle the level of difficulty of question 7.

1 2 3 4 5 6 7 8 9

8. Graph $y = x + 1$ (1 point)



Circle the level of difficulty of question 8.

1 2 3 4 5 6 7 8 9

9. Refer to problem 8. What does the following a, b, and c need to be so that the line and parabola intersect? You may use different color pens/pencils to sketch the parabolas onto the prior graph.

a) y = ax^2 + 2: a = _____

(2 points)

Circle the level of difficulty of question 9a.

1 2 3 4 5 6 7 8 9

b) y = x^2 + bx: b = _____

(2.5 points)

Circle the level of difficulty of question 9b.

1 2 3 4 5 6 7 8 9

c) y = x^2 + c: c = _____

(2 points)

Circle the level of difficulty of question 9c.

1 2 3 4 5 6 7 8 9

Appendix C – Mid-Test

Thesis Mid-Test

Part 1 – Concrete, 1-2 dimensions

Name _____

Refer to these descriptions when answering the "level of difficulty" problems.

1 = Very Easy 2 = Easy 3 = Average 4 = Challenging but Manageable
5 = Challenging but Barely Manageable 6 = Difficult 7 = Very Difficult
8 = It's so hard I don't know where to begin but I'll try to write something. 9 = I give up!

Show Your Work

1. Graph $3x + 5 \geq x$

(2 points each answer)

Interval Notation: _____ Set-Builder Notation: _____

Circle the level of difficulty of question 1.

1 2 3 4 5 6 7 8 9

2. Graph $-3x \geq 9$

(2 points)

Circle the level of difficulty of question 2.

1 2 3 4 5 6 7 8 9

3. Are the above two graphs discrete or continuous sets? (1 point) _____

How do you know? (1 point) _____

Circle the level of difficulty of question 3.

1 2 3 4 5 6 7 8 9

4. Find the intersection and union. (1 point each answer)

$\{a, b, c\} \cap \{a, x\}$ = _____

$\{a, b, c\} \cup \{a, x\}$ = _____

Circle the level of difficulty of question 4.

1 2 3 4 5 6 7 8 9

5. Are the above two sets discrete or continuous? (1 point) _____

How do you know? (1 point) _____

Circle the level of difficulty of question 5.

1 2 3 4 5 6 7 8 9

6. What is an example of an equation or inequality in 1-dimension? (1 point)

_____

Circle the level of difficulty of question 6.

1 2 3 4 5 6 7 8 9

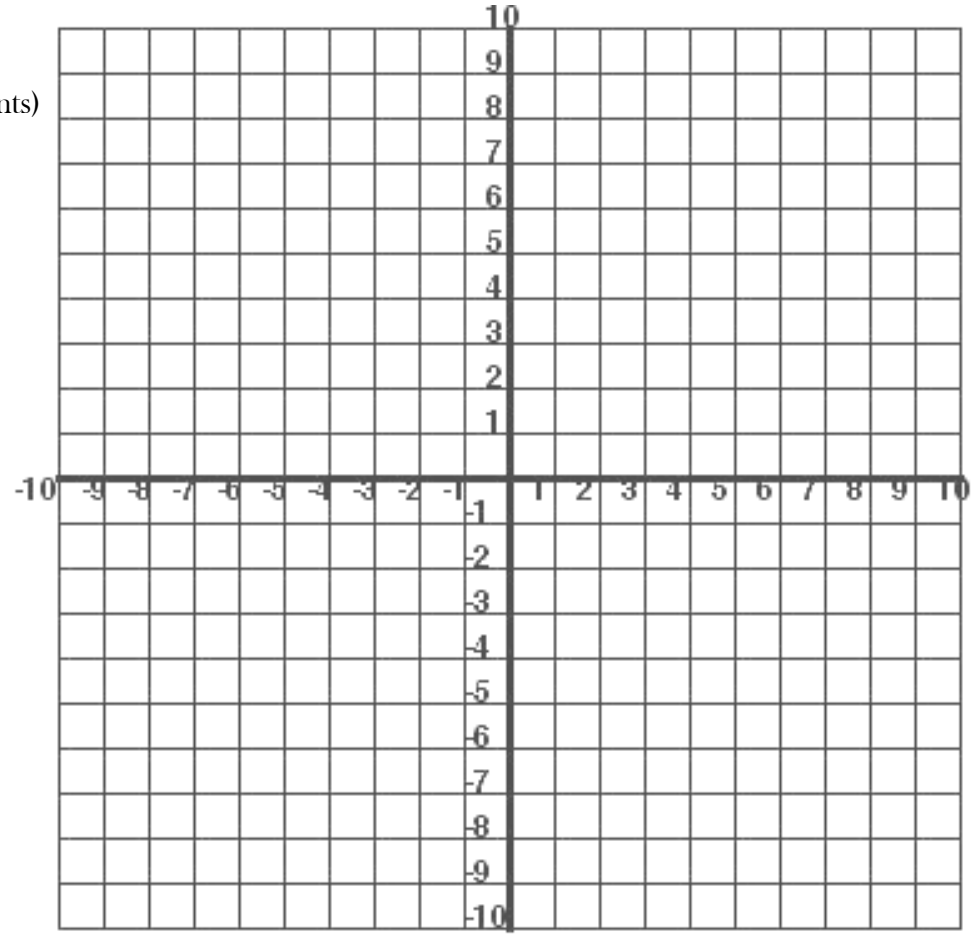
7. What is an example of an equation or inequality in 2-dimensions? (1 point)

_____

Circle the level of difficulty of question 7.

1 2 3 4 5 6 7 8 9

8. Graph $y < 2x - 1$ (3 points)



Circle the level of difficulty of question 8.

1 2 3 4 5 6 7 8 9

Appendix D – Post-Test

Thesis Post Test

Part 2 – Abstract, 2-3 dimensions

Name _____

As in part 1, refer to these descriptions when answering the "level of difficulty" problems.

1 = Very Easy 2 = Easy 3 = Average 4 = Challenging but Manageable
5 = Challenging but Barely Manageable 6 = Difficult 7 = Very Difficult
8 = It's so hard I don't know where to begin but I'll try to write something. 9 = I give up!

Show Your Work

1. For all x, $x^2 \geq -a^2$. True False

Why? (2.5 points)

_____

_____

_____

_____

Circle the level of difficulty of question 1.

1 2 3 4 5 6 7 8 9

2. If αx $> a^2$ then x $> \alpha$. Is this always true? Explain your answer. (3 points)

_____

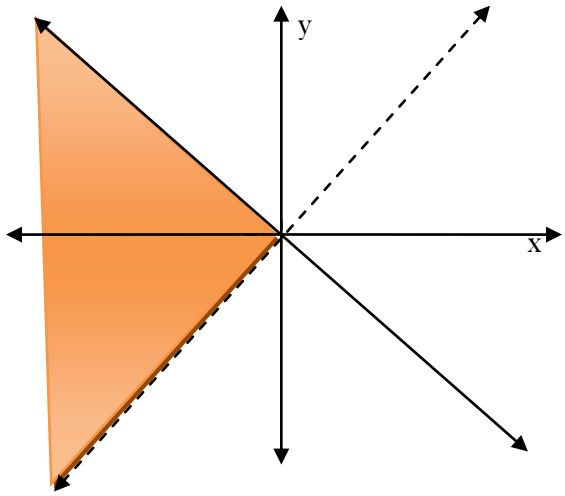
_____

_____

Circle the level of difficulty of question 2.

1 2 3 4 5 6 7 8 9

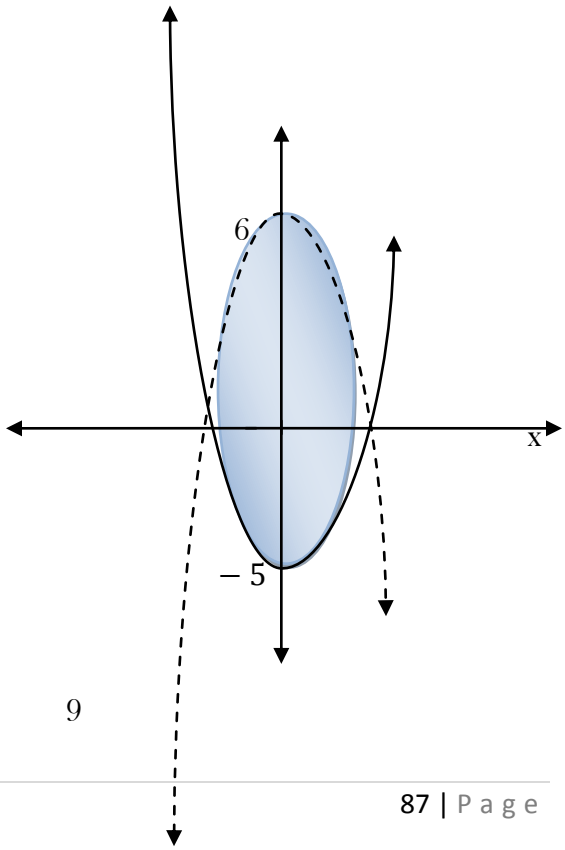
3. What is the system of inequalities that represents this graph? (3 points)



Circle the level of difficulty of question 3.

1 2 3 4 5 6 7 8 9

4. What is the system of inequalities that represents this graph? (3 points)

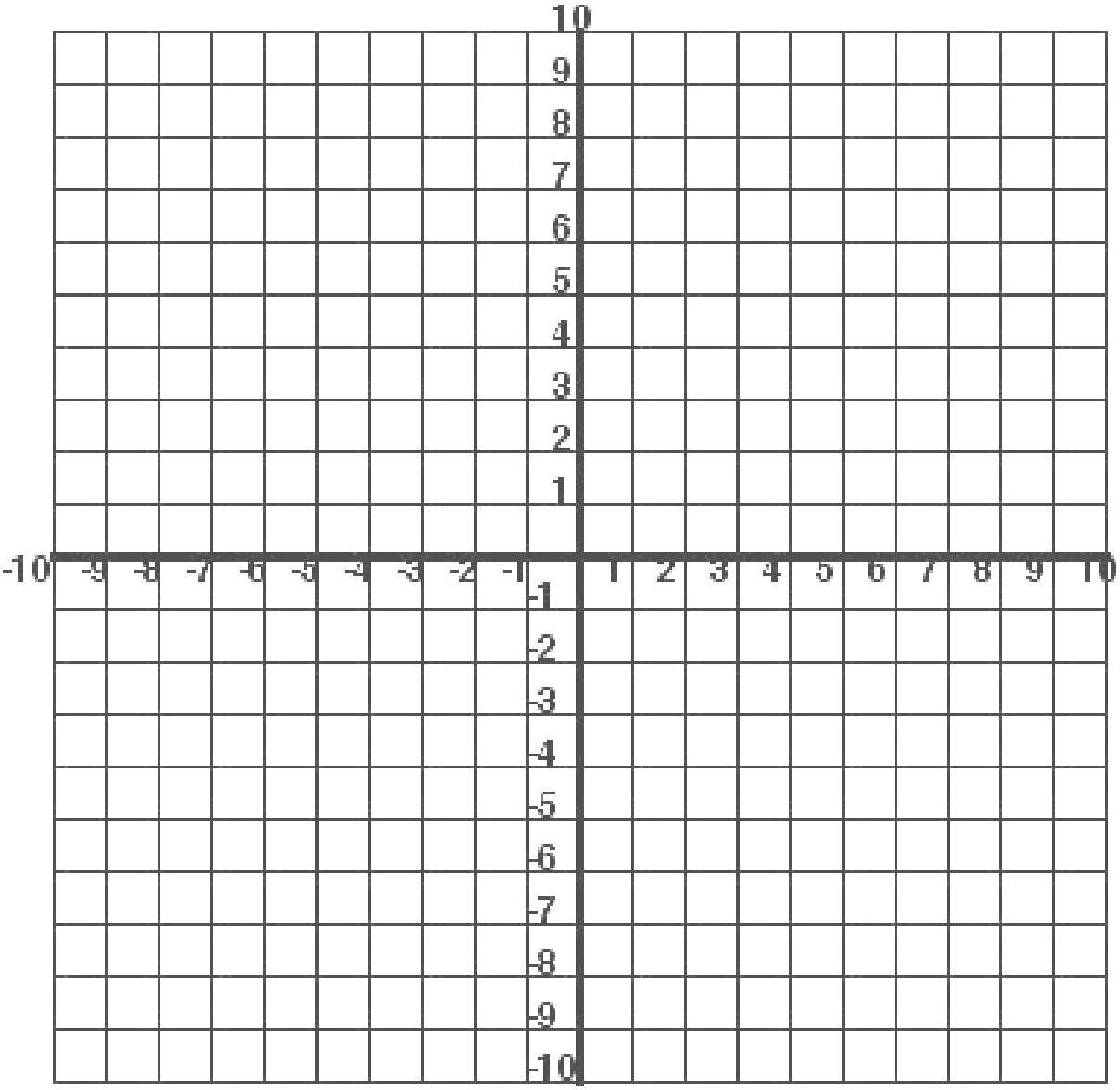


Circle the level of difficulty of question 4.

1 2 3 4 5 6 7 8 9

5. Graph the system of inequalities. Find the coordinates of any vertices (corners) formed. Make sure your graphs are as clear and accurate as possible. (8 points)

$-4x + 2y - 8 \geq 0$ $\quad -y > 2x$ $\quad x > -5$ $\quad y \geq 0$



( ), ( ), ( ), and ( )

Circle the level of difficulty of question 5.

1 2 3 4 5 6 7 8 9

6. What is an example of an equation or inequality in 2-dimensions? (1 point)

_____

Circle the level of difficulty of question 6.

1 2 3 4 5 6 7 8 9

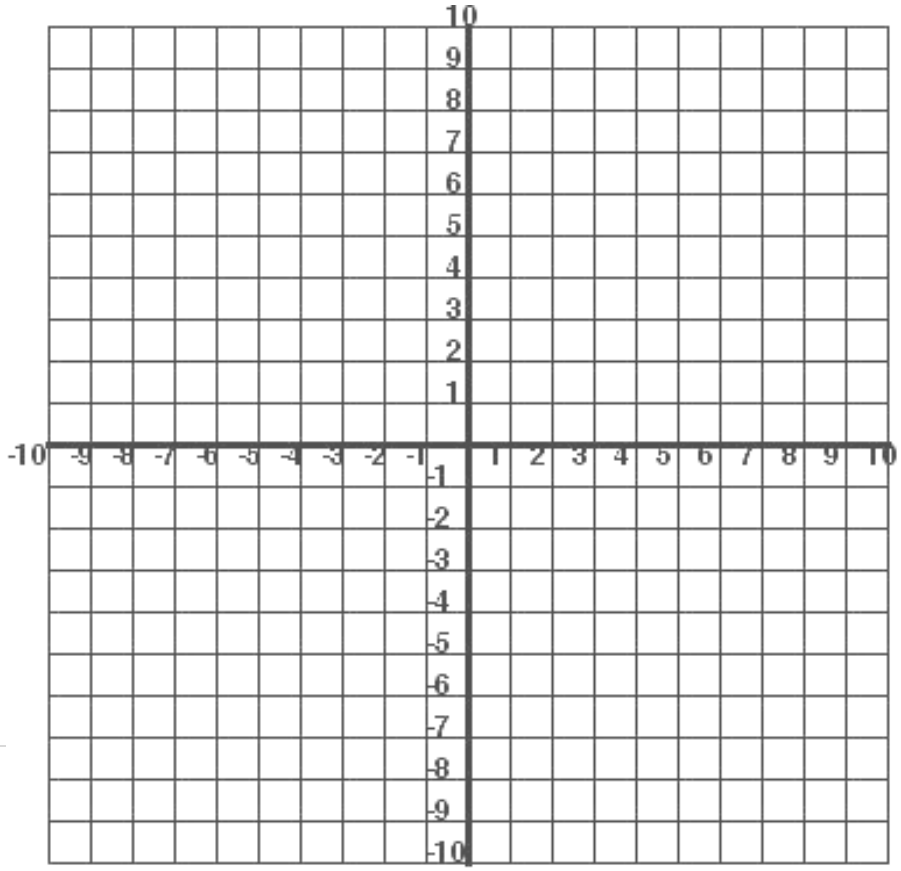
7. What is an example of an equation or inequality in 3-dimensions? (2 points)

_____

Circle the level of difficulty of question 7.

1 2 3 4 5 6 7 8 9

8. Graph $y = x + 1$ (1 point)



Circle the level of difficulty of question 8.

1 2 3 4 5 6 7 8 9

9. Refer to problem 8. What does the following a, b, and c need to be so that the line and parabola intersect? You may use different color pens/pencils to sketch the parabolas onto the prior graph.

a) y = ax^2 + 2: a = _____

(2 points)

Circle the level of difficulty of question 9a.

1 2 3 4 5 6 7 8 9

b) y = x^2 + bx: b = _____

(2.5 points)

Circle the level of difficulty of question 9b.

1 2 3 4 5 6 7 8 9

c) y = x^2 + c: c = _____

(2 points)

Circle the level of difficulty of question 9c.

1 2 3 4 5 6 7 8 9

Appendix E – Activity 1: Confidence Level Questionnaire

Activity 1: Discrete Sets

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Venn Diagrams

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Sets

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Unions and Intersections

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Word Problems

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Venn Diagrams

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Sets

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Unions and Intersections

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Word Problems

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What part of this activity was most helpful in your understanding? Least helpful?

_____

Appendix F – Activity 1: Post Activity Test

Post-Activity 1: Discrete Sets

Name _____

1. Find the intersection and union. (2 point each for a and b, 3 points for c)

a) {x, y, 2, 5, 7z} ∩ {2, x, y, ☺} = _____

b) {x, y, 2, 5, 7z} ∪ {2, x, y, ☺} = _____

c) Draw a Venn diagram that represents the two sets above. Label them set A and set B. Now draw set C = {1, 3}

For questions 3 and 5: Think of an example that we haven't talked about in class.

2. How would you describe a discrete set? (1 point)

_____

_____

3. What's an example of a discrete set? (2 points)

_____

_____

Appendix G – Activity 2: Confidence Level Questionnaire

Activity 2: Skewers and Money

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Understanding Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Solving for Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing Inequalities on a Number Line

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities and Intersections

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and Unions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Understanding Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Solving for Inequalities

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing Inequalities on a Number Line

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities and Intersections

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and Unions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What part of this activity was most helpful in your understanding? Least helpful?

_____

Appendix H – Activity 2: Post Activity Test

Post-Activity 2: Continuous Sets

Name _____

1 point each answer.

1. How would you describe a continuous set?

_____

_____

2. What's an example of a continuous set?

_____

_____

3. $a \leq 2$ OR $a \geq 2$

a) Graph:

b) Is this a union or intersection?

_____

c) How do you know?

_____

_____

_____

4. $a \leq 2$ AND $a \geq 2$

a) Graph:

b) Is this a union or intersection?

_____

c) How do you know?

_____

_____

_____

5. What's the difference between a union and an intersection?

_____

_____

_____

6. Think about your major, what job you have or hope to have some day, or an event in your life. Think of an example where inequalities can be used to give you info to help you make a decision. For example, I'm thinking about buying a car. I've discovered that the interest rate for a car loan depends on my credit score, my age, income, employment stability, and loan amount. To get a low interest rate, my age needs to be between 25 and 58 years, I need to be employed at the same company for at least one year, I can't have a negative action on my credit report, etc.. Meaning, I want to be in the intersection of each criteria so that I can get the lowest interest rate possible.

Appendix I – Activity 3: Confidence Level Questionnaire

Activity 3: Playdough

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Graphing in 2-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and graphing in 2-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Graphing in 2-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and graphing in 2-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What part of this activity was most helpful in your understanding? Least helpful?

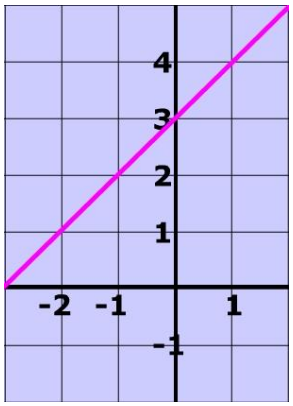
_____

Appendix J – Activity 3: Post Activity Test

Post-Activity 3: Playdough

Name _____

2 points for problem 1, 4, and 5; 1 point for each additional answer.



1. What is the equation of the above line?

_____

2. Now change the = to ≥ . Write inequality here:

_____

3. Change the graph to look like the inequality in problem 2 (Write directly on the graph above).

4. Name two points that are included in the inequality. ( ) ( )

5. Name two points that are NOT included in the inequality. ( ) ( )

6. What happens if the inequality is changed from ≥ to >?

_____

_____

_____

7. What happens if the inequality is changed from $\geq$ to $\leq$?

_____

_____

_____

Appendix K – Activity 4: Confidence Level Questionnaire

Activity 4: Transparencies

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Understanding Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Solving for Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities and Intersections in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and Unions in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Understanding Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Solving for Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Graphing Inequalities in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities and Intersections in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Inequalities and Unions in 2 dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What is the hardest thing about inequalities for you? The easiest?

_____

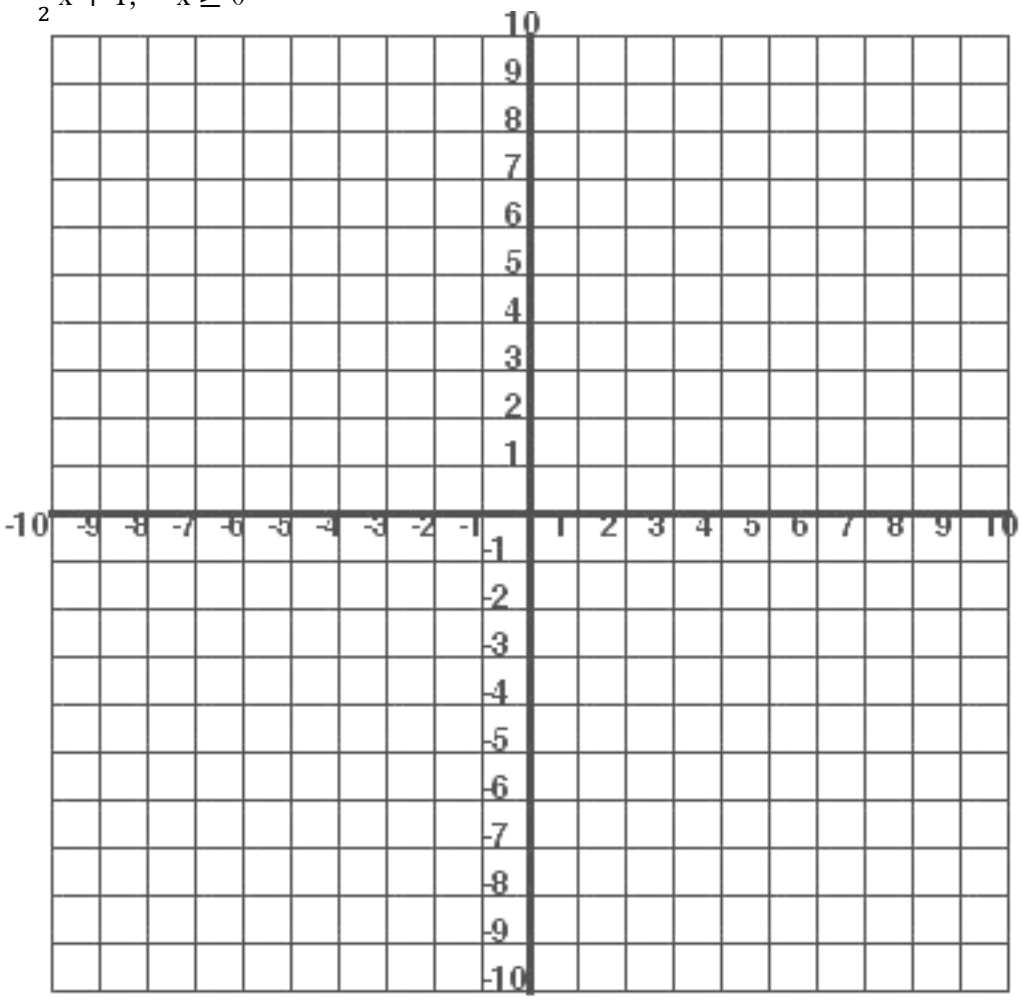
Appendix L – Activity 4: Post Activity Test

Post-Activity 4: Transparencies

Name _____

5 points for problem 1; 2 points each for problem 2 and 3; 1 point for problem 4

1. Graph $y \geq x - 6$, $y < -\frac{1}{2}x + 1$, $x \geq 0$



2. Name two points that are included in this graph. ( ) ( )

3. Name two points that are NOT included in this graph. ( ) ( )

4. What was the hardest part about graphing problem 1?

_____

_____

_____

Appendix M – Activity 5: Confidence Level Questionnaire

Activity 5: Graph Blast Bingo

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Finding an equation to represent a 1-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Finding an equation to represent an inequality

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Intersections in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Finding an equation to represent a 1-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Finding an equation to represent an inequality

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Intersections in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What part of this activity was most helpful in your understanding? Least helpful?

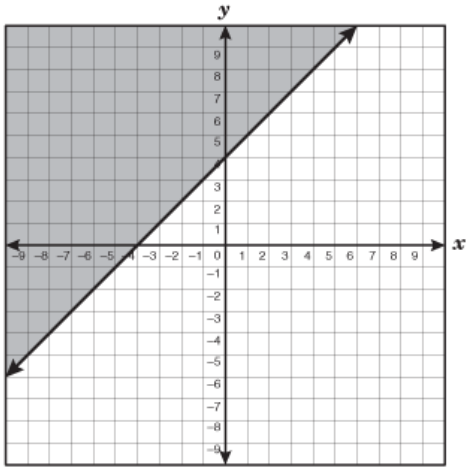
_____

Appendix N – Activity 5: Post Activity Test

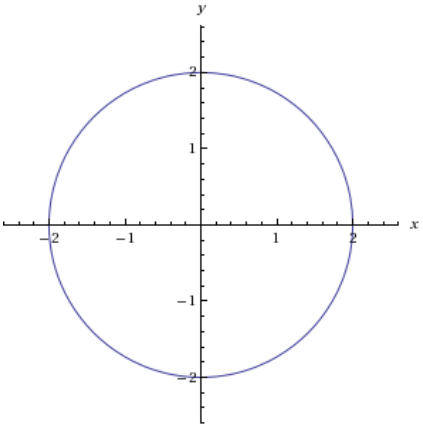
Post-Activity 5: Graph Blast Bingo

Name _____

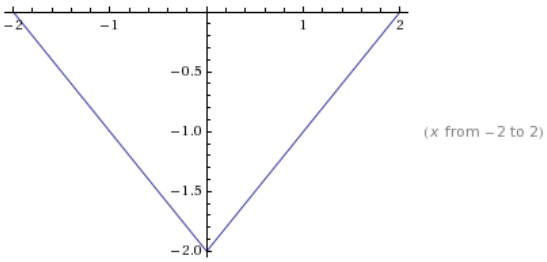
Write the equations that represents the following graphs. (1 point per problem)

1. 

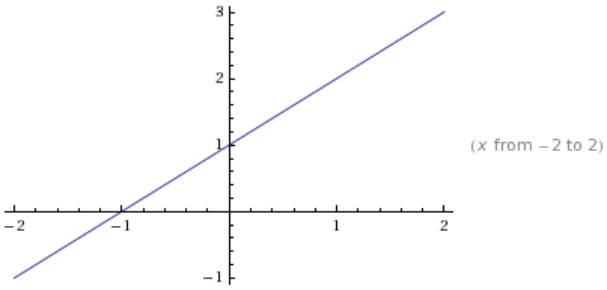
_____

2. 

_____

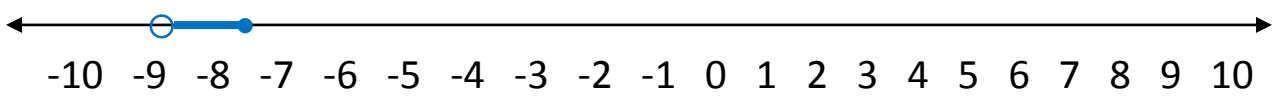
3. 

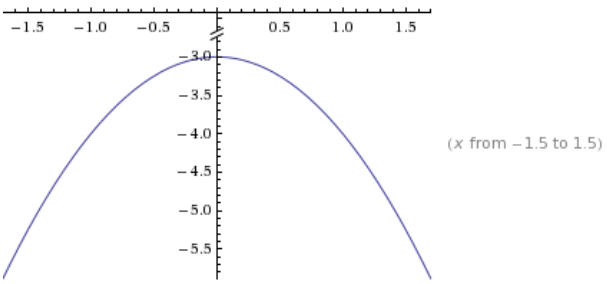
_____

4. 

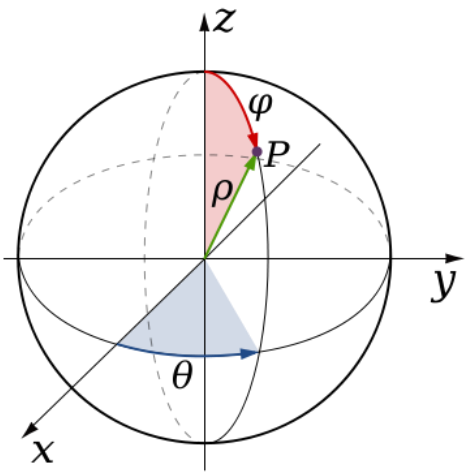
_____

5. (See number line below) _____

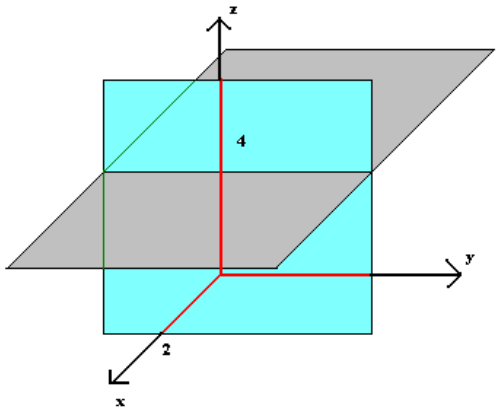


6. 

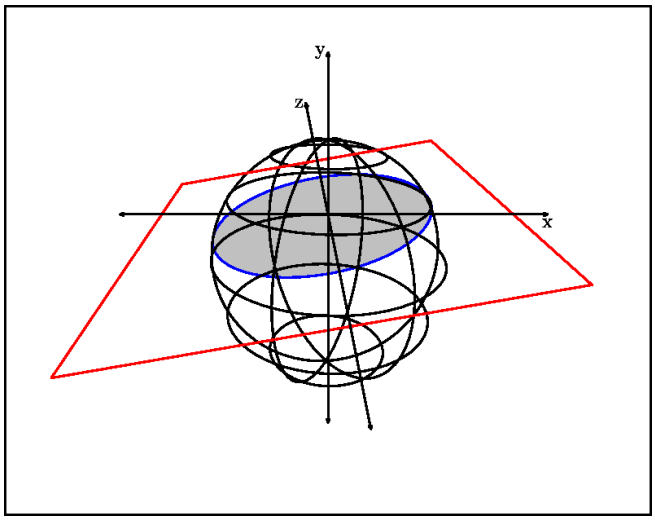
_____

7. 

_____

8. 

_____

9. 

Write an equation that will represent the shaded (gray) area of this sphere and plane. To do this, come up with two equations (one that will represent the sphere and another to represent the plane) and then write them as an intersection.

_____

10. a) What does the intersection of a sphere and plane create? What dimension is it in?

_____

b) Now I want greater than or equal to the plane. What will we have? What dimension is it in?

_____

_____

Appendix O – Activity 6: Confidence Level Questionnaire

Activity 6: Aquarium

Name _____

How do you feel **BEFORE** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Understanding 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Intersections in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

How do you feel **AFTER** the activity about the following concepts? (Circle your answer and/or write comments.)

1. Finding an equation to represent a 2-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

2. Understanding 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

3. Finding an equation to represent a 3-dimensional graph

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

4. Inequalities in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

5. Intersections in 3-dimensions

1=Very Confident 2=Confident 3=Slightly Confident 4=Neutral 5=Slightly Unsure 6=Unsure 7=Very Unsure 8=I felt totally lost. 9=Other (explain):

_____

6. What part of this activity was most helpful in your understanding? Least helpful?

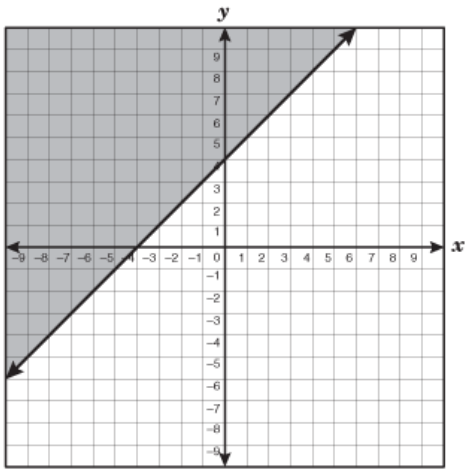
_____

Appendix P – Activity 6: Post Activity Test

Post-Activity 6: Aquarium

Name _____

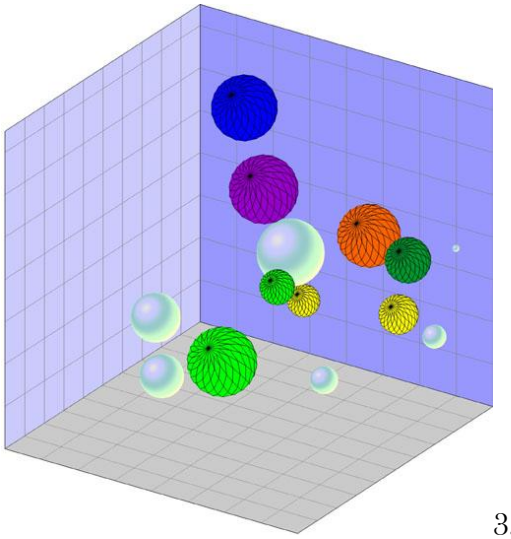
(1 point for each answer)



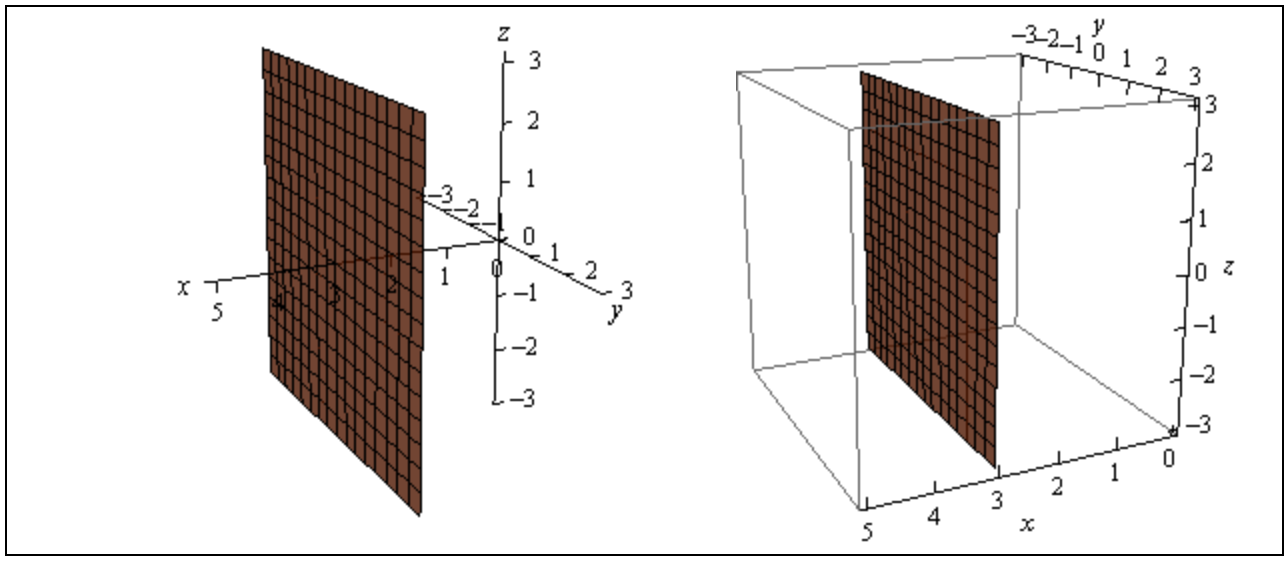
1. Write the equations that represents the graph. _____

2. The above graph is obviously in 2-dimensions. What would the equation be in 3-dimensions? Hint: Does adding thickness change the equation?

_____

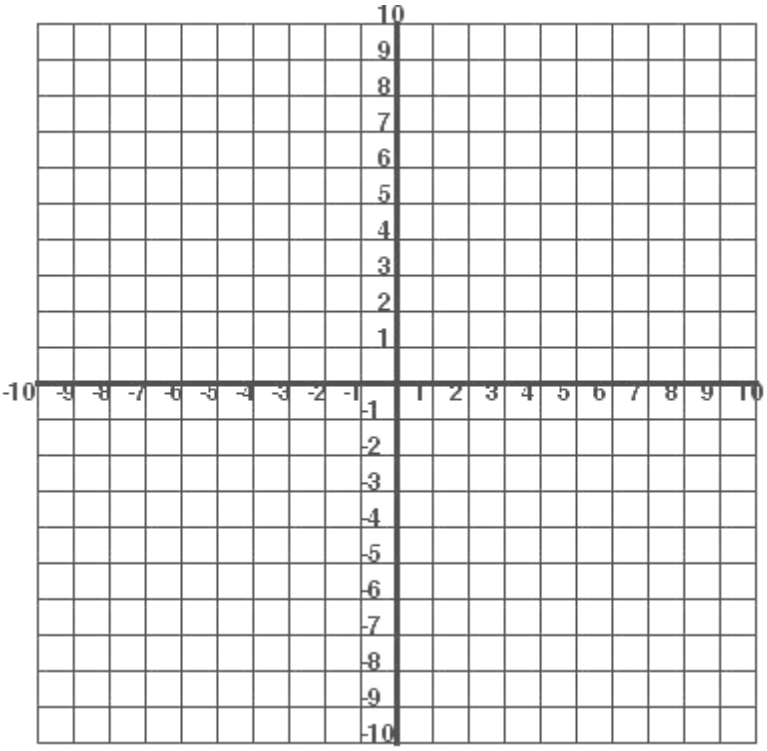


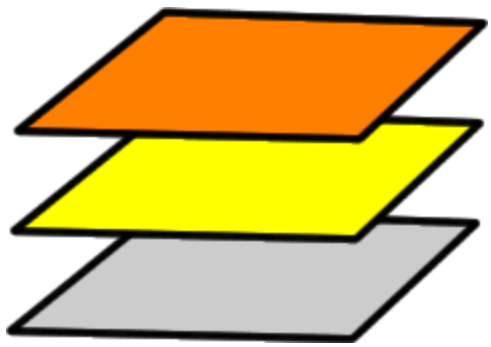
3. How many sets are in this cube? _____



4. This is the same graph, one drawn on a traditional coordinate system, the other drawn inside a box. What is the equation of this plane? _____

5. Draw the above graph in 2 dimensions.





6. What might the equations be for these three planes?

_____

_____

_____

7. For all x, $x^2 \geq -a^2$. If x is a variable and a is any constant, will the right side always be negative or positive? _____

8. What if I said αx $> a^2$ then x $> \alpha$. If x is a variable and a is any constant, prove to me that this is not correct. Meaning, when will x not be greater than α? Think about both sides when evaluating the problem.

_____

_____

_____

Appendix Q – Student Info Questions A

Student Info Questions A

Name _____

1. I hate math. T F

2. Most of the time, math is not too bad. T F

3. My major is _____.

4. What teaching ideas do you have that would make learning more enjoyable and help you retain knowledge? _____

_____

_____.

5. I am a visual learner. T F

6. Working on the board with another person, rather than alone, will be okay. T F

7. My career goal after I graduate college is to _____.

8. One of my favorite things to do is _____.

9. I am willing to work hard to succeed. T F

10. My strengths are _____.

11. If I had to choose, I prefer **math theory** / **applied math**.

12. I want this class to be fun and fair. T F

13. I have taken this class _____ time(s) before.

14. I have taken the ELM. T F My score: _____.

15. I learn best when: I'm rewarded for effort I memorize I study alone
I study with others I talk about the material with someone who knows more than me
I'm encouraged I'm challenged I apply the material to something I understand
the learning environment is casual the learning environment is formal
listening to music it's quiet I learn short cuts to understand even if they are silly

Other: _____

16. Write any comments, concerns, or suggestions on back of page.

Appendix R – Student Info Questions B

Student Info Questions B

Name _____

1. What was the last math class you took? _____

2. What letter grade did you earn in that class? _____

3. How much time does it typically take you to finish a math homework assignment for this class?

1 – 30 minutes 30 – 60 minutes 1 – 1 ½ hours 1 ½ - 2 hours 2 – 4 hours 4+ hours

4. Are you the first one in your family to attend college? Y N

5. Are you a morning person? Y N

6. My high school GPA is _____.

7. How many units are you taking? _____

8. Are you involved in any clubs/groups? Y N

9. In a week, about how much time do you spend reading/studying Math 95 (excluding homework)?

1 – 30 minutes 30 – 60 minutes 1 – 1 ½ hours 1 ½ - 2 hours 2 – 4 hours 4+ hours

10. When working on Math 95 outside of class, do you mostly work alone? Y N

11. Do you have a tutor for this class? Y N

12. I have a job that requires me to work _____ hours a week.

13. Your stress level this semester: Very Low Low Medium High Very High

Thank you!! ☺

Appendix S – Activity 5: Bingo Cards

This is a sampling of what I used in this study.

Graph Blast Bingo

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$x = 2$ and $z = 4$	$[-4, \infty)$ or $\{x \mid x \geq -4\}$
$y < 2x + 1$	$y = x^2$

Graph Blast Bingo

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$(-\infty, -2.5) \cup [1, \infty)$ or

$\{x \mid -\infty < x < -2.5 \text{ or } 1 \leq x < \infty\}$

$y = (1/25)\, x^2 + 5$

$x^2 + y^2 + z^2 = \left(\frac{d}{2}\right)^2$

$x^2 + y^2 = 1$

Graph Blast Bingo

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$[-6, \infty) \cap (-\infty, 1)$ or
$\{x \mid -6 \leq x < \infty \text{ and } -\infty < x < 1\}$

$y = x^2 - 1$

$x^2 + y^2 = 1$

$[-4, \infty)$ or $\{x \mid x \geq -4\}$

Graph Blast Bingo

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$x^2 + y^2 = 9$ and $z = 3$	$\{x \mid x = 6.5\}$
$[-6, \infty) \cap (-\infty, 1)$ or $\{x \mid -6 \leq x < \infty \text{ and } -\infty < x < 1\}$	$y = x^2 - 1$

Graph Blast Bingo

0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$y < 2x + 1$

$\{x \mid x \text{ is } \mathbb{R} \text{ and } x \neq -3, 0\}$

$y = x^2$

$x^2 + y^2 + z^2 = \rho^2$

Graph Blast Bingo

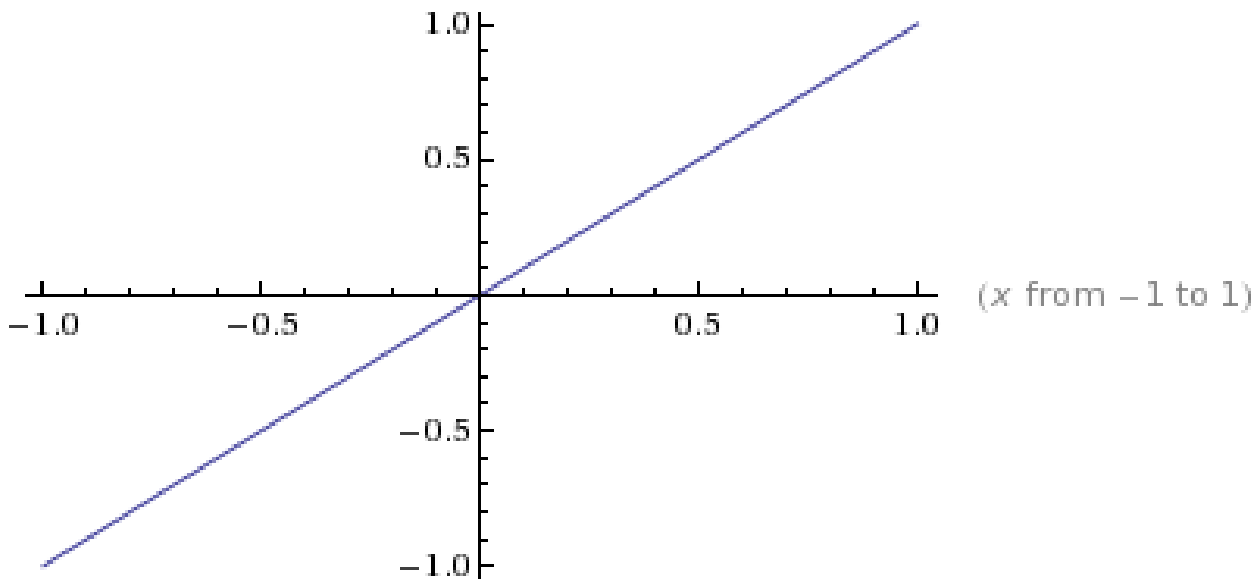
0-D Pt	1-D Num Line	2-D Line	2-D Parabola	2-D Absolute Value	2-D Polynomial	2-D Circle	2-D Inequalities	3-D Plane	3-D Sphere

$y = x - 1$	$\{x \mid x \text{ is } \mathbb{R} \text{ and } x \neq -0.5\}$
$y \geq x + 4$	$x^2 + y^2 + z^2 = \rho^2$

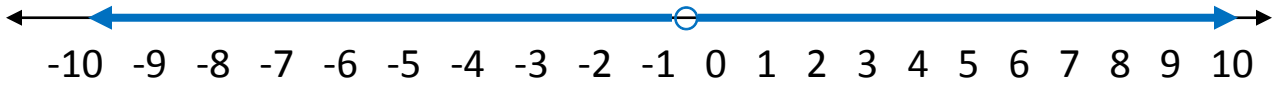
Appendix T – Activity 5: Bingo Game

This is a sampling of what I used in this study.

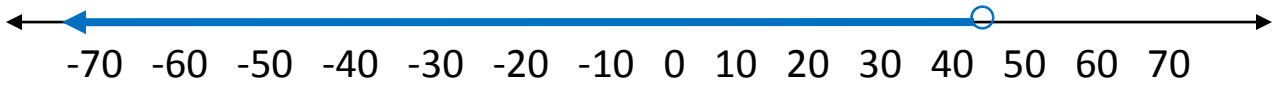
1



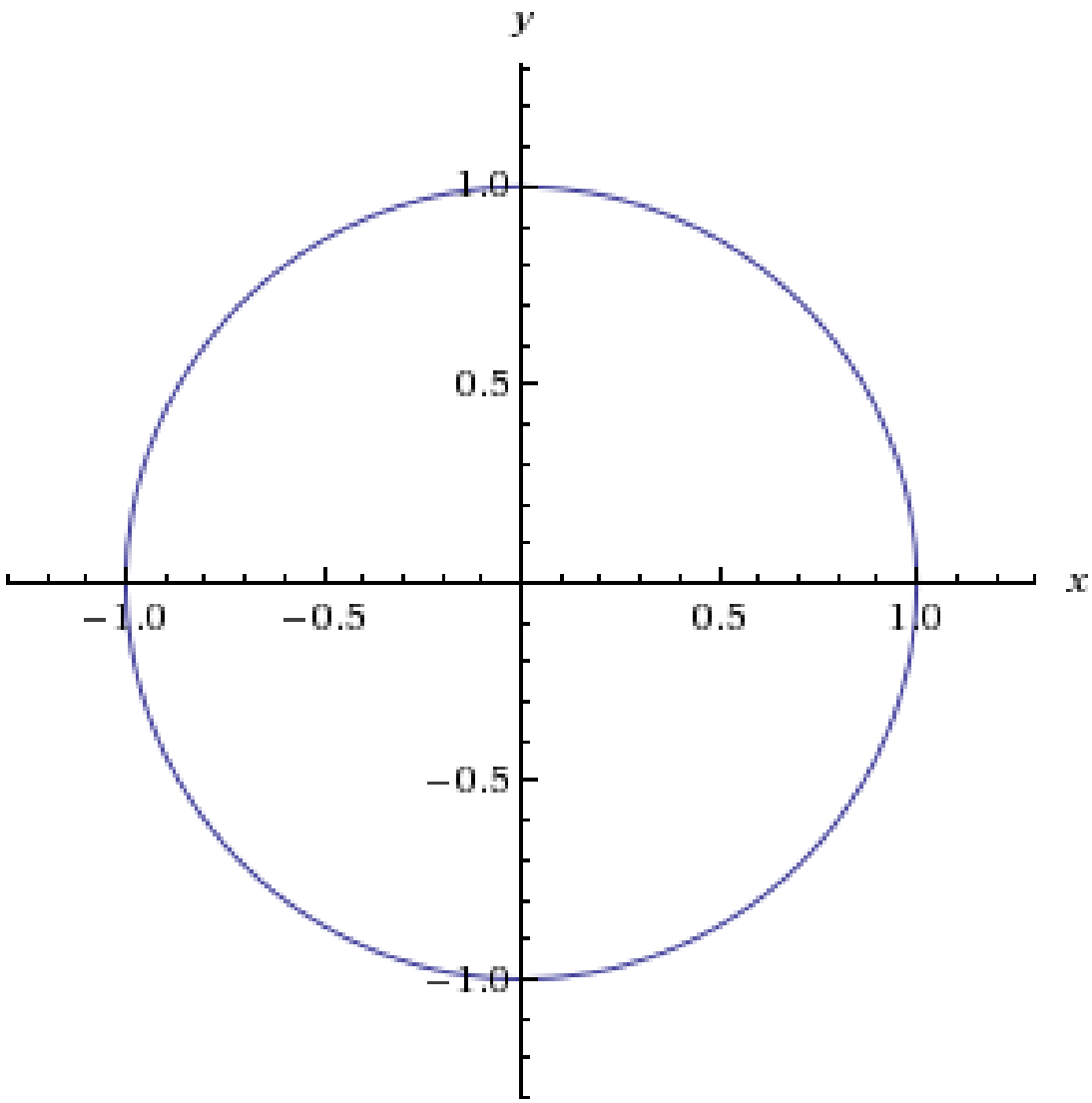
2



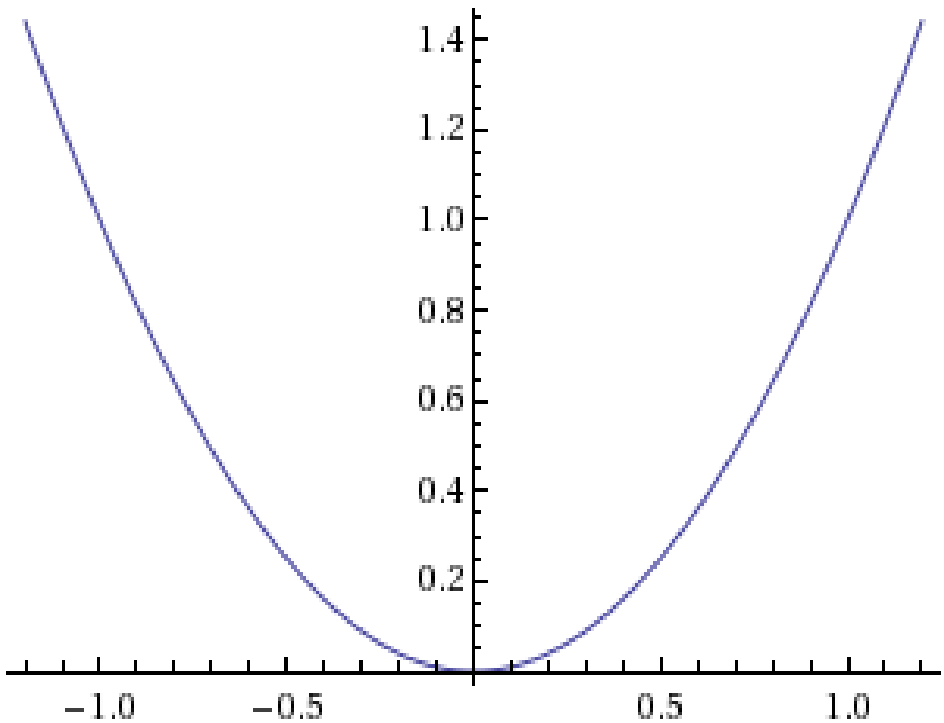
3



4

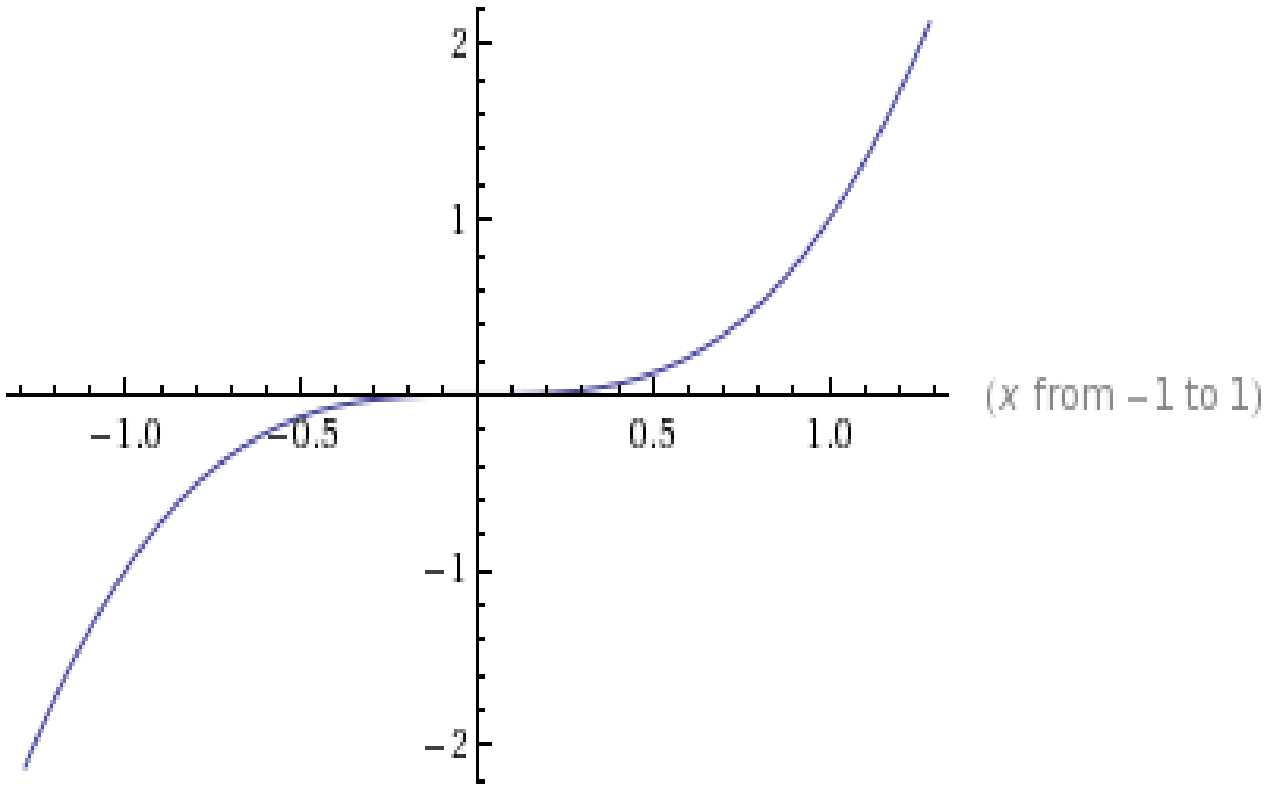


5

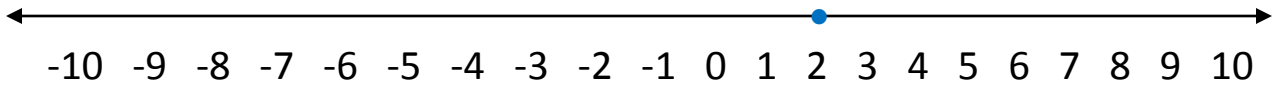


(x from -1 to 1)

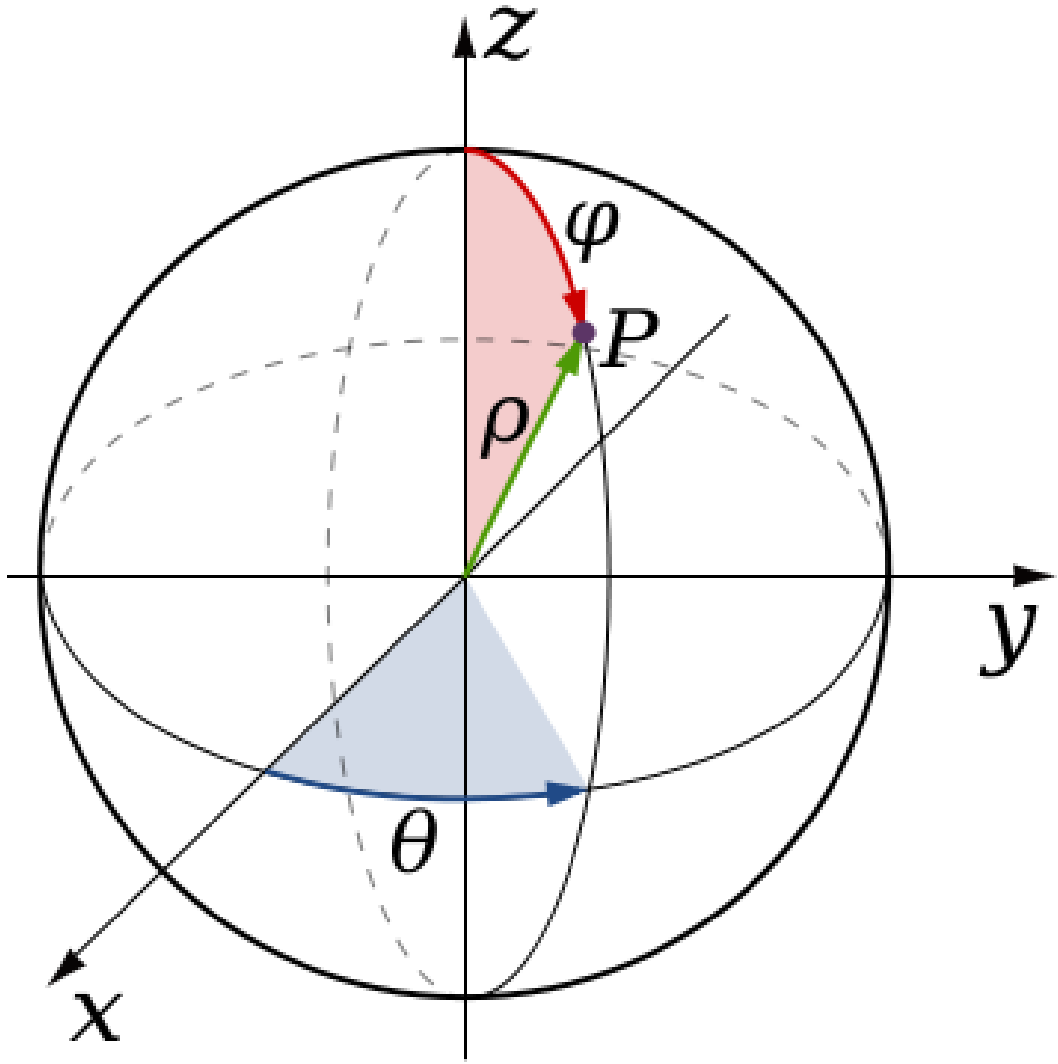
10



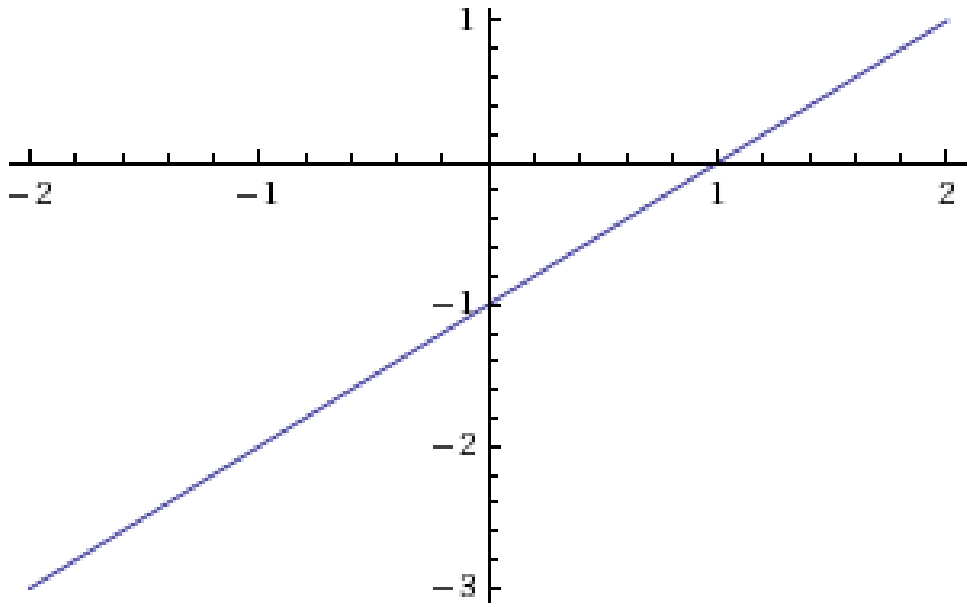
11



12

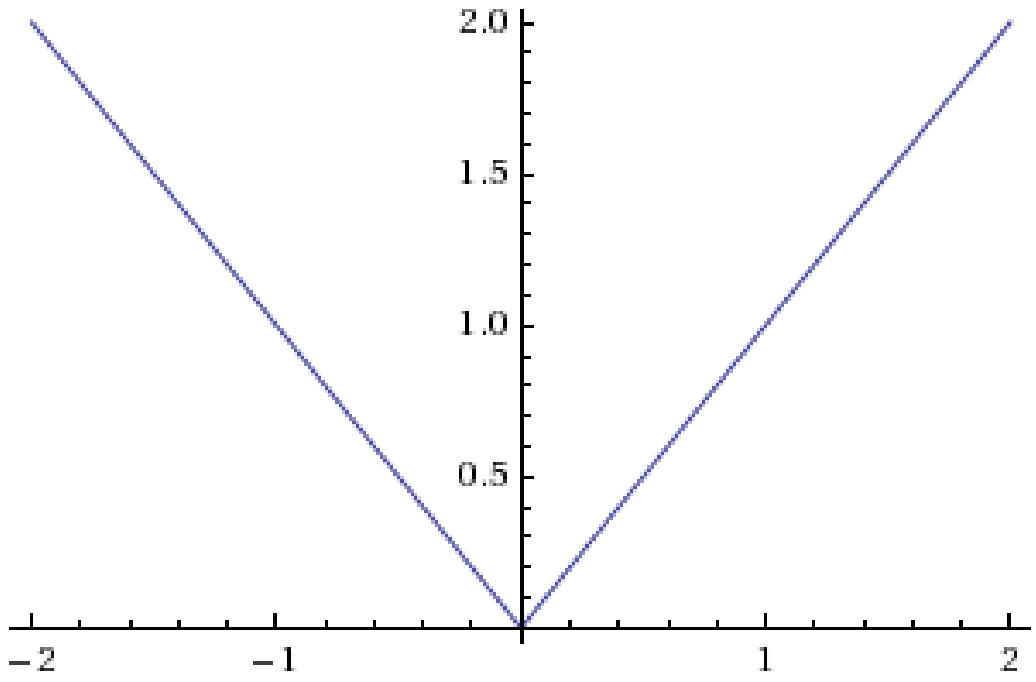


13



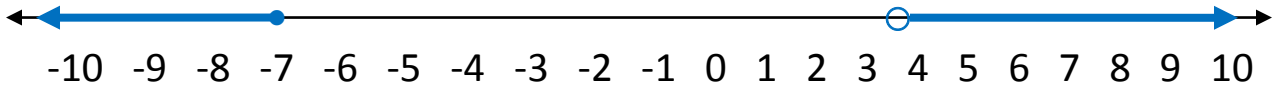
(x from -2 to 2)

15



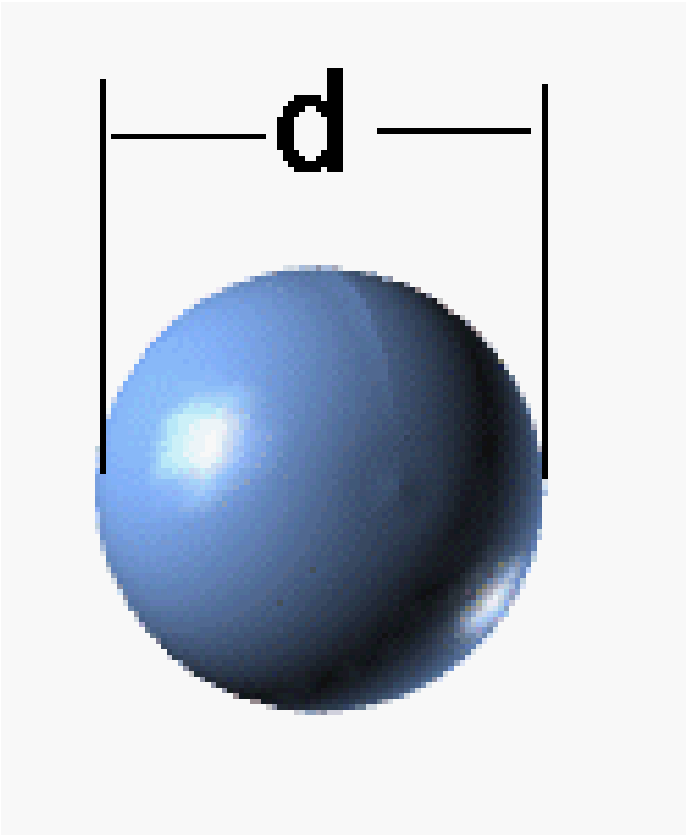
(x from -2 to 2)

17

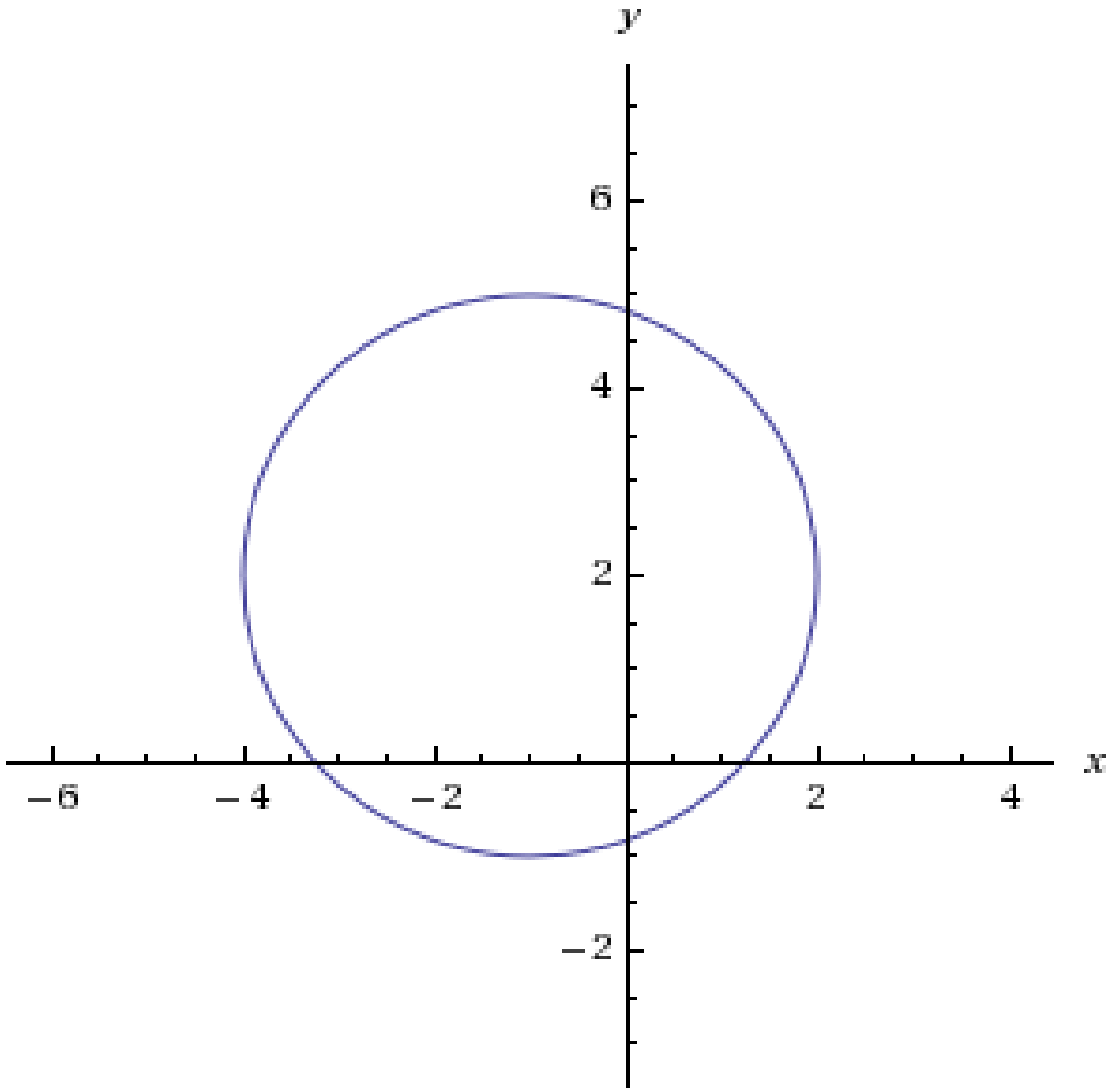


18

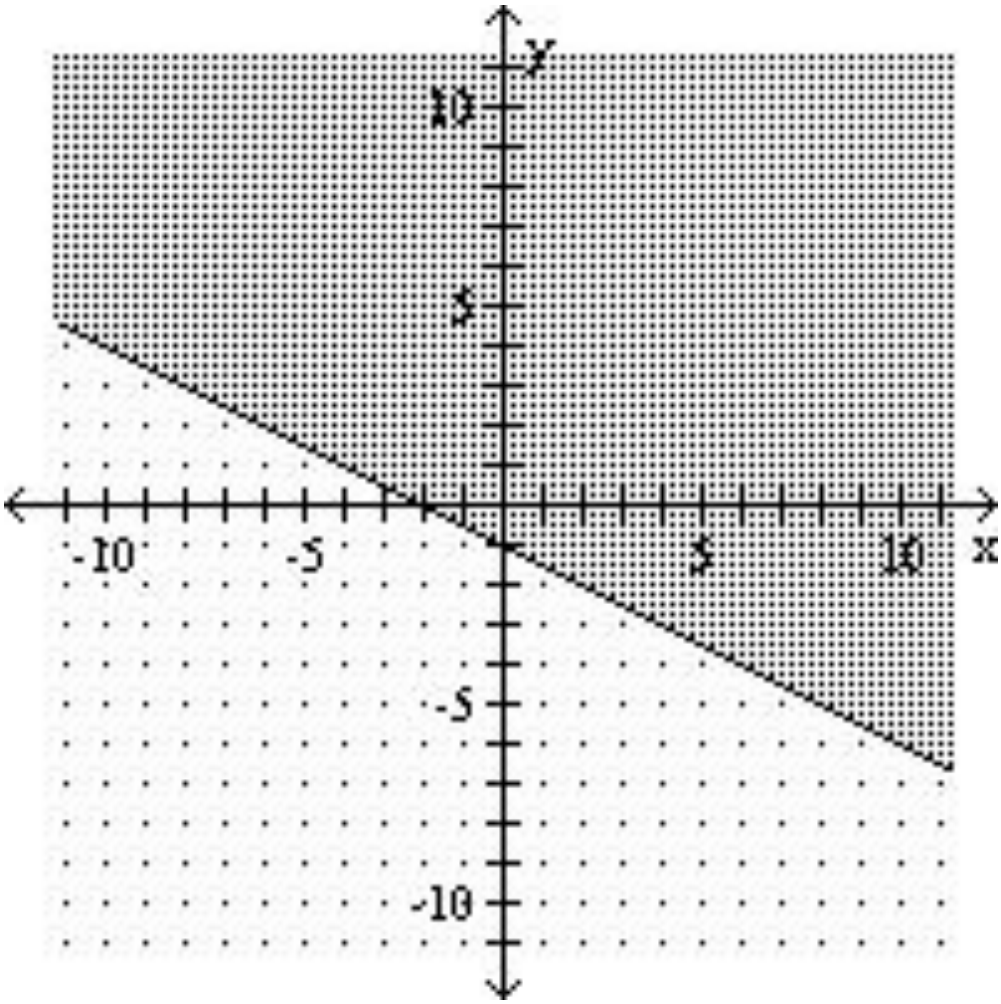
Let the origin be (0, 0, 0).



19



21



Appendix U – Samples of Raw Data

I learn best when I

Student	Hates Math	Freshman	Taken Class Before	study alone	study with others	talk w/ someone more knowledgable	Prior Class	Grade Earned in Prior Class	Grade Translated to Number	Enrolled in math class after study?	What class enrolled in?	Major Before	Major After	First in Family to Attend College	Units Currently Taking	Jobs Hours per Week	Involved in Clubs/Groups	Tutor	Morning Person	Mostly Work Alone	Stress Level this Semester	High School GPA
1	N	Y	N	N	N	Y	PC	C	2	N		Undeclare	Undeclare	N	13	0	Y	N	N	Y	M	3.1
2	Y	Y	Y	N	N	Y	PC	B	3	N		Art	Computer	N	14	0	N	N	N	Y	M	3.25
3	Y	Y	N	N	Y	N	APC	C-	2	N		Biology	Biology	N	14	0	N	N	N	N	VH	3.5
4	Y	Y	N	N	Y	Y	A2	C+	2.5	Y	MT	Art	Art	N	14	0	Y	Y	Y	Y	VL	3.6
5	N	Y	N	Y	Y	Y	T	C	2	N		Business	Business	Y	12	0	Y	N	N	Y	M	3.2
6	N	Y	N	N	N	N	A2	B-	3	N		History	History	N	13	15	N	N	N	Y	VH	3.7
7	Y	Y	N	N	N	N	S	C	2	Y	MT	History	Computer	Y	12	0	Y	N	N	Y	H	3.4
8	N	Y	N	N	N	Y	MA	B	3	N		Biology	Undeclare	N	14	0	Y	N	Y	Y	M	3.2
9	N	Y	Y	N	N	Y	A2	D	1	Y	CA	Liberal St	Liberal St	Y	12	0	N	N	Y	Y	VL	3.1
10	Y	Y	Y	N	Y	Y	A2	D	1	Y	CA	Art	Art	Y	5	20	N	N	Y	Y	VH	2
11	Y	Y	N	N	Y	N	PC	C	2	N		Biology	Biology	N	13	10	Y	Y	N	N	M	2.8
12	N	Y	N	N	Y	Y	A2	B	3	N		Pre-Nursi	Pre-Nursi	N	15	12	N	N	N	Y	M	3.67
13	Y	Y	Y	N	Y	Y	A2	C	2	N		Pre-Nursi	Pre-Nursi	N	12	0	Y	Y	N	Y	M	3
14	N	Y	N	Y	N	N	PC	B+	3.5	Y	BC, MTM	Business	Business	N	14	12	Y	N	N	Y	H	3.5
15	N	Y	Y	Y	N	N	PC	C	2	N		Undeclare	Undeclare	Y	12	0	N	Y	Y	Y	M	3.5
16	N	Y	N	N	N	N	DM	B	3	Y	CA	Liberal St	Liberal St	N	15	35+	N	Y	N	N	VH	3.8
17	N	Y	N	N	N	N	C	D+	1.5	N		Pre-Nursi	Pre-Nursi	N	17	0	N	Y	N	Y	H	3.5
18	Y	Y	N	Y	N	N	PC	C	2	N		Pre-Nursi	Pre-Nursi	Y	13	20	N	N	N	Y	M	3
19	N	Y	N	Y	N	Y	MA	N/A	N/A	Y	PC	Undeclare	Undeclare	N	14	0	N	N	N	N	M	3.4
20	Y	Y	N	N	N	N	MM	A	4	Y	MT	Undeclare	Undeclare	N	12	18	N	N	N	N	M	3.4
21	Y	Y	N	N	N	N	S	B	3	Y	M95	Communi	Communi	N	14	0	Y	N	N	Y	H	3.5
22	N	Y	Y	Y	N	N	S	B	3	Y	M95	Politcal Sc	History	N	14	0	Y	N	N	Y	L	3.6
23	Y	Y	N	N	N	Y	SAP	B	3	Y	PC, MTM	Undeclare	Undeclare	N	12	0	N	N	Y	Y	M	3.5
Total Students		23																				

Student	Pre-Test - Part 1 Confidence Level									Pre-Test - Part 2 Confidence Level										Mid-Test Confidence Level							
	Problem1	Problem2	Problem3	Problem4	Problem5	Problem6	Problem7	Problem8	Problem9	Problem1	Problem2	Problem3	Problem4	Problem5	Problem6	Problem7	Problem8	Problem9a	Problem9b	Problem9c	Problem1	Problem2	Problem3	Problem4	Problem5	Problem6	Problem7	Problem8
1	3	2	3	8	8	6	7	6	4	8	8	8	6	8	8	8	8	8	8	8	3	1	4	3	3	3	5	3
2	9	2	9	9	9	9	9	9	7	9	9	9	9	7	7	7	6	9	9	9	6	3	4	6	6	3	3	3
3	8	9	8	9	9	6	6	9	8	8	9	9	9	9	8	9	9	8	9	9	4	1	4	3	4	3	5	9
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17	2	2	6	6	6	2	7	5	6	6	6	6	6	6	6	6	4	6	4	6	2	2	2	4	4	3	5	3
18	6	2	3	8	6	9	9	3	3	8	7	9	8	7	9	9	5	9	9	9	4	1	4	2	5	3	4	2
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20	5	4	3	9	9	6	7	7	2	8	8	8	8	5	9	9	1	8	8	8	6	3	1	8	8	1	1	1
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22	8	2	7	9	9	4	9	9	9	3	2	9	9	9	9	9	9	9	9	9	2	2	2	2	2	2	2	3
23	5	4	6	9	9	9	9	9	6	8	8	9	9	8	9	9	5	8	9	9	8	4	4	4	8	4	9	5

Pretest

Part 1

		Problems													
		1a	1b	1c	2	3a	3b	4a	4b	5a	5b	6	7	9	Total
	Key	1	2	3	4	5	6	7	8	9	10	11	12	13	Ref
Allen Sales	AlSa	1	0	0	1	1	0	0	0	0	0	0	0	0	3.00
Anthony DiMizio	AnDi	0	0	0	1	0	0	0	0	0	0	0	0	0.5	1.50
Ashley Loera	AsLo	0	0	0	0	0	0	0	0	0	0	1	0	0	1.00
Ben Blanchard	BeBl	0	0	0	0	0	0	0	0	0	0	0	1	0	1.00
Elizabeth Corona	ElCo	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
Greg Amantia	GrAm	2	0	0	2	1	0	0	0	0	0	0	1	0.75	6.75
Greg Lomeli	GrLo	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
"Haiti" Jehiry DeHaro	JeDe	0	0	0	0	0	0	1	0	1	0	0	0	0	2.00
Hannah Dineleya	HaDi	2	0	0	0.5	0	0	0	0	0	0	1	0	0.5	4.00
Jacqueline Taylor	JaTa	0	0	0	2	1	0	0	0	0	0	0	0	0	3.00
"Jazmin" Azucena Rocha	JaRo	1.25	0	0	1.25	1	0	0	0	0	0	1	0	0	4.50
Katie Sharp	KaSh	1.5	0	0	2	0	0	0	0	0	0	0	0	1	4.50
Katie Smolarski	KaSm	0	0	0	0.5	1	0	0	0	0	0	0	0	0.5	2.00
Krista Simmons	KrSi	0	0	0	0.5	1	0	0	0	1	0	1	1	1.5	6.00
Laura Flores	LaFl	0	0	0	1	0	0	0	0	0	0	1	0	0	2.00
Laura Gentry	LaGe	0	0	0	1.25	0	0	0	0	0	0	0	0	0.75	2.00
Laura Morales	LaMo	1	0	0	1	0	0	0	0	0	0	1	0	0	3.00
Lauren Creager	LaCr	0.5	0	0	0.5	0	0	1	0.5	1	0	0	0	1.5	5.00
Lauren Mickeal	LaMi	0	0	0	0	1	0	0	0	1	0.5	1	0	0	3.50
Molly Ellena	MoEl	1	0	0	2	1	0	0	0	0	0	0	0	1	5.00
Stephanie Benedetti	StBe	0	0	0	0.25	1	0	0	0	0	0	0	0	0.5	1.75
Tom Reese	ToRe	0	0	0	2	1	0	0	0	0	0	0	0	0	3.00
Wendy Menchaca	WeMe	2	0	0	2	1	0	0	0	0	0	0	0	0	5.00
Totals		2	2	2	2	1	1	1	1	1	1	1	1	3	19
															3.021739

Mid-Test

		Problems													
		1a	1b	1c	2	3a	3b	4a	4b	5a	5b	6	7	8	Total
	Key	1	2	3	4	5	6	7	8	9	10	11	12	13	Ref
1	AlSa	2	0	0	2	1	1	1	1	1	1	1	1	2.25	14.25
2	AnDi	2	0	2	1	1	0	0	0	1	1	1	1	2	12.00
3	AsLo	2	0	0	2	1	0	1	0.5	1	0	1	0	0	8.50
4	BeBl	2	0	0	1.5	0	0	1	1	0	0	1	0	1.5	8.00
5	ElCo	2	0	0	1	1	0	1	1	1	0	1	1	0.5	9.50
6	GrAm	2	0	1.75	2	1	0	0	1	1	0	1	1	1.5	12.25
7	GrLo	0	0	0	1.5	1	0	0	0	1	1	1	1	1.5	8.00
8	JeDe	1	0	0	2	1	1	1	1	1	1	1	0	1.5	11.50
9	HaDi	2	0	0	2	1	0	1	1	1	1	1	1	1.5	12.50
10	JaTa	0	0	0	2	1	0	0	1	1	0	0	0	1	6.00
11	JaRo	0.5	0	0	2	1	0	1	1	1	1	1	0	1	9.50
12	KaSh	1.75	0	1	1.5	1	1	1	1	1	0	1	0	3	13.25
13	KaSm	0	0	0	2	1	0	0	0	1	0	0	0	2	6.00
14	KrSi	0	0	0	1	1	1	1	1	1	1	0	1	1	9.00
15	LaFl	1	0	0	2	0	0	0	1	0	0	1	0	1	6.00
16	LaGe	1.5	0	1.5	2	1	0	1	0.75	1	0	1	0	1.5	11.25
17	LaMo	1.75	0	0	2	1	1	0	1	1	0	1	0	2.5	11.25
18	LaCr	1.5	0.5	0.25	1.5	1	0	1	1	1	0	0	1	3	11.75
19	LaMi	2	0	2	1	1	0	0	0	1	0	0	1	1.5	9.50
20	MoEl	0	0	0	2	1	1	0	0	0	0	1	0	3	8.00
21	StBe	0	0	0	1	1	1	1	0.75	1	1	0	0	2.5	9.25
22	ToRe	2	0	0	2	1	1	1	0.75	1	1	0	1	1.5	12.25
23	WeMe	1.5	0	0	2	1	0	1	1	1	0	0	0	1.5	9.00
Totals		2	2	2	2	1	1	1	1	1	1	1	1	3	19
															9.934783

Data Spreadsheets - Microsoft Excel

A1 Total Scores

Total Scores	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Total Pre-Tests	3	4.5	1	4.5	1	7.75	0	5.25	7	4	7.5	16	6	13	3	4	8	14.5	12.25
Mid/Post-Tests	31.25	30	23.25	25.5	17.25	27.25	16.25	23.5	31	12	24.75	21.5	16.5	24.75	19	24.25	17.25	24.75	20
Difference (Post - Pre)	28.25	25.5	22.25	21	16.25	19.5	16.25	18.25	24	8	17.25	5.5	10.5	11.75	16	20.25	9.25	10.25	7.75

Sample Size		23		
Correlation		0.17		
				Scores
Mean	Difference	16.96	Pre	6.33
			Post	23.28
Standard Deviation	Difference	6.31	Pre	4.24
			Post	5.42
Mode	Difference	16.25	Pre	4.50
			Post	24.75
Median	Difference	17.3	Pre	5.3
			Post	23.5
Highest Score	Difference	28.25	Pre	16
			Post	32.5
Lowest Score	Difference	5.5	Pre	0
			Post	12

Null Hypothesis: $H_0: \mu_d \leq 0$. The mean of the Post-Test/Pre-Test differences is less than or equal to 0.

Alternate Hypothesis: $H_a: \mu_d > 0$. The mean of the Post-Test/Pre-Test differences is greater than 0.

t-Test: α = 0.05; t = 12.895; p-value = 0.0000

Reject the null hypothesis? Yes: there is evidence that the null hypothesis if false.

Results - Total Scores Pre1 Mid | Results - Total Scores Pre2 Pos | Results - Total Scores | Results - Total Scores (2) | Results - Subject Discrete | Results - Subject Continuous | Results - Subject Inequalities | Results - Subject 0, 1 D | Results - Subject 2 D | Results - Subject 3 D

Data Spreadsheets - Microsoft Excel

A27 Total Points

Data Analysis by Subject

	Discrete Sets				Continuous Sets				Inequalities				0, 1 D				2 D	
Total Score	Pre	Mid/Post	Part 1/Mid Increase	Percent Increase	Pre	Mid/Post	Part 1/Mid Increase	Percent Increase	Pre	Mid/Post	Pre-tests/Mid/Post	Percent Increase	Pre	Mid/Post	Pre-tests/Mid/Post	Percent Increase	Pre	Mid/Pos
1	0	4	4.00	100%	3	6	3.00	30%	2	18.75	16.75	52%	2	9	7.00	48%	0	16
2	0	2	2.00	50%	1	6	5.00	50%	1.5	15.5	14.00	43%	1	6	5.00	34%	1.5	1
3	0	2.5	2.50	63%	0	5	5.00	50%	0	17	17.00	52%	1	7	6.00	41%	0	12
4	0	2	2.00	50%	0	3.5	3.50	35%	1.5	10.5	9.00	28%	1.5	6.5	5.00	34%	2	
5	0	3	3.00	75%	0	4	4.00	40%	0	8.75	8.75	27%	0	4	4.00	28%	1	8
6	0	2	2.00	50%	5	6.75	1.75	18%	5.75	15.75	10.00	31%	5	9.75	4.75	33%	1.75	1
7	0	2	2.00	50%	0	2.5	2.50	25%	0	9.75	9.75	30%	0	5.5	5.50	38%	0	7
8	2	4	2.00	50%	0	5	5.00	50%	3.25	11.5	8.25	25%	3.25	5.5	2.25	16%	0	
9	0	4	4.00	100%	2.5	5	2.50	25%	6	17	11.00	34%	6.5	7	0.50	3%	0.5	1
10	0	2	2.00	50%	3	3	0.00	0%	2	6.5	4.50	14%	2	4	2.00	14%	1	
11	0	4	4.00	100%	3.5	3.5	0.00	0%	3	10.5	7.50	23%	3.5	3.5	0.00	0%	3	14
12	0	3	3.00	75%	3.5	6.25	2.75	28%	15	13.5	-1.50	-5%	3.5	5.75	2.25	16%	12.5	10
13	0	1	1.00	25%	1.5	3	1.50	15%	4.5	8.5	4.00	12%	4	3	-1.00	-7%	1	1
14	1	4	3.00	75%	1.5	3	1.50	15%	7	12.75	5.75	18%	1.5	4	2.50	17%	9.5	14
15	0	1	1.00	25%	1	3	2.00	20%	1	13	12.00	37%	2	6	4.00	28%	1	1
16	0	2.75	2.75	69%	1.25	6	4.75	48%	3.5	15.5	12.00	37%	1.25	7	5.75	40%	2.75	1
17	0	2	2.00	50%	2	5.75	3.75	38%	6.5	10.25	3.75	12%	6	6.75	0.75	5%	2	
18	2.5	3	0.50	13%	1	4.75	3.75	38%	4	15.25	11.25	35%	2	5.75	3.75	26%	10	1
19	1.5	1	-0.50	-13%	1	6	5.00	50%	3	13	10.00	31%	2	8.5	6.50	45%	5.75	
20	0	0	0.00	0%	4	4	0.00	0%	6	22	16.00	49%	3	4	1.00	7%	4	
21	0	3.75	3.75	94%	1.25	3	1.75	18%	2.75	12.75	10.00	31%	0.25	3.75	3.50	24%	3	
22	0	3.75	3.75	94%	3	6	3.00	30%	3.5	14.75	11.25	35%	3.5	4	0.50	3%	0	13
23	0	3	3.00	75%	5	4.5	-0.50	-5%	5	14.5	9.50	29%	5	4.5	-0.50	-3%	0.5	1
Total Points	4	4			10	10			32.5	32.5			14.5	14.5			28.5	2
Mean	0.30	2.60	2.29	57%	1.91	4.59	2.67	27%	3.77	13.36	9.59	29%	2.60	5.68	3.09	21%	2.73	12
Standard Dev	0.72	1.16	1.27	32%	1.58	1.34	1.75	18%	3.23	3.61	4.38	13%	1.83	1.83	2.39	17%	3.49	4
Low	0	0	-0.50	-13%	0	2.5	-0.50	-5%	0	6.5	-1.50	-5%	0	3	-1.00	-7%	0	
High	2.5	4	4.00	100%	5	6.75	5.00	50%	15	22	17.00	52%	6.5	9.75	7.00	48%	12.5	

Questions	Pre-Test 1	Pre-Test 2	Mid-Test	Post-Test
Discrete Sets	4, 5		4, 5	
Continuous Sets	1, 2, 3		1, 2, 3	
Inequalities	1, 2, 9	1, 2, 3, 4, 5	1, 2, 8	1, 2, 3, 4, 5

General Info | Post Activity Tests | By Question | Pre-Test - Part 1 | Pre-Test - Part 2 | Mid-Test | Post-Test | By Subject | Data Analysis - by Tests | Data Analysis - Chart | Data Analysis - By Subject | Data Analysis - by Subject (2) | Results - Total Scores Pre1 Mid | Results - Total Sc